

10 Meetkunde met vectoren

Voorkennis Afstanden en middens

Bladzijde 52

1 $P(\frac{1}{2}(5+11), \frac{1}{2}(-1+5)) = P(8, 2)$

$$Q(\frac{1}{2}(11+3), \frac{1}{2}(5+7)) = Q(4, 6)$$

$$d(A, P) = \sqrt{(8-5)^2 + (2-(-1))^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$d(A, Q) = \sqrt{(4-5)^2 + (6-(-1))^2} = \sqrt{(-1)^2 + 7^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$d(P, Q) = \sqrt{(4-8)^2 + (6-2)^2} = \sqrt{(-4)^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

De omtrek van driehoek APQ is $3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$.

2 a $p = 3$ geeft $A(-2, 3)$, $B(4, 5)$ en

$$d(A, B) = \sqrt{(4-(-2))^2 + (5-3)^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}.$$

b $\frac{1}{2}(-2+p+1) = -6$

$$-2+p+1 = -12$$

$$p = -11$$

$p = -11$ geeft $A(-2, -11)$, $B(-10, 5)$ en

$$d(A, B) = \sqrt{(-10-(-2))^2 + (5-(-11))^2} = \sqrt{(-8)^2 + 16^2} = \sqrt{64+256} = \sqrt{320} = 8\sqrt{5}.$$

c $M(\frac{1}{2}(-2+p+1), \frac{1}{2}(p+5)) = M(\frac{1}{2}p - \frac{1}{2}, \frac{1}{2}p + 2\frac{1}{2})$

M op $y = -x$ geeft $\frac{1}{2}p + 2\frac{1}{2} = -(\frac{1}{2}p - \frac{1}{2})$

$$\frac{1}{2}p + 2\frac{1}{2} = -\frac{1}{2}p + \frac{1}{2}$$

$$p = -2$$

$p = -2$ geeft $A(-2, -2)$, $M(-1\frac{1}{2}, 1\frac{1}{2})$ en

$$d(A, M) = \sqrt{(-1\frac{1}{2}-(-2))^2 + (1\frac{1}{2}-(-2))^2} = \sqrt{(\frac{1}{2})^2 + (3\frac{1}{2})^2} = \sqrt{\frac{1}{4} + 12\frac{1}{4}} = \sqrt{12\frac{1}{2}} = 2\frac{1}{2}\sqrt{2}.$$

d $d(A, B) = \sqrt{(p+1-(-2))^2 + (5-p)^2} = \sqrt{(p+3)^2 + (5-p)^2} = \sqrt{p^2 + 6p + 9 + 25 - 10p + p^2}$
 $= \sqrt{2p^2 - 4p + 34}$

e $d(A, B) = 8$ geeft $\sqrt{2p^2 - 4p + 34} = 8$

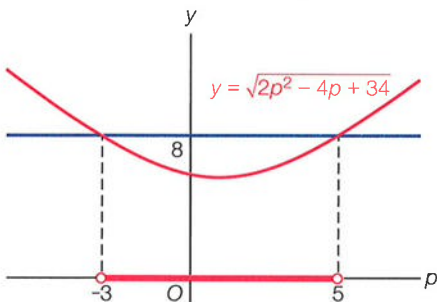
$$2p^2 - 4p + 34 = 64$$

$$2p^2 - 4p - 30 = 0$$

$$p^2 - 2p - 15 = 0$$

$$(p+3)(p-5) = 0$$

$$p = -3 \vee p = 5$$



Dus $d(A, B) < 8$ voor $-3 < p < 5$.

10.1 Vectoren

Bladzijde 53

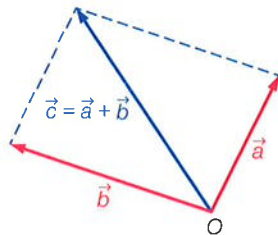
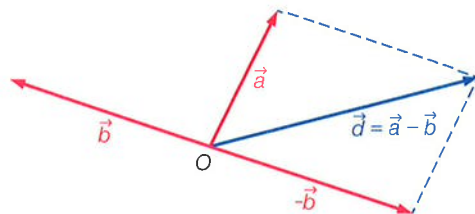
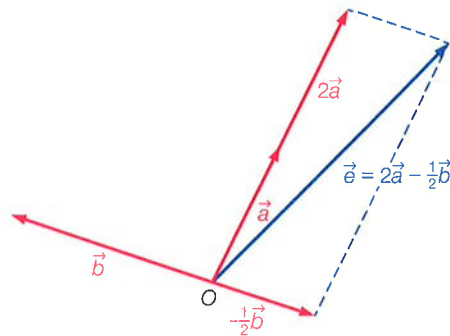
1 $d(O, A) = \sqrt{(5-0)^2 + (2-0)^2} = \sqrt{5^2 + 2^2} = \sqrt{29}$

$$d(A, B) = \sqrt{(2-5)^2 + (6-2)^2} = \sqrt{(-3)^2 + 4^2} = 5$$

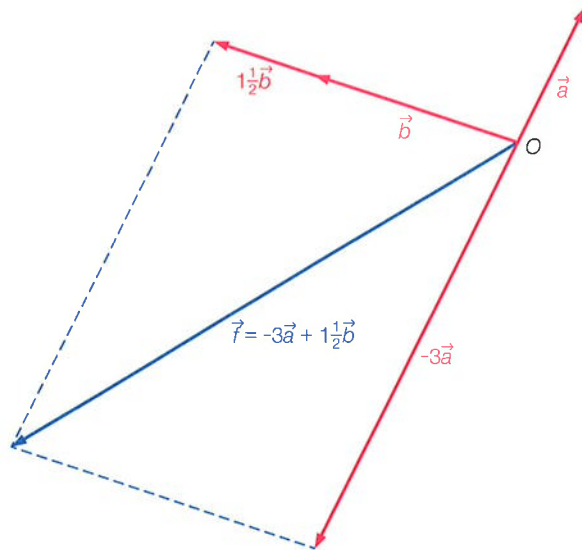
Dus de lengte van de wandeling is $5 + \sqrt{29}$.

Bladzijde 56

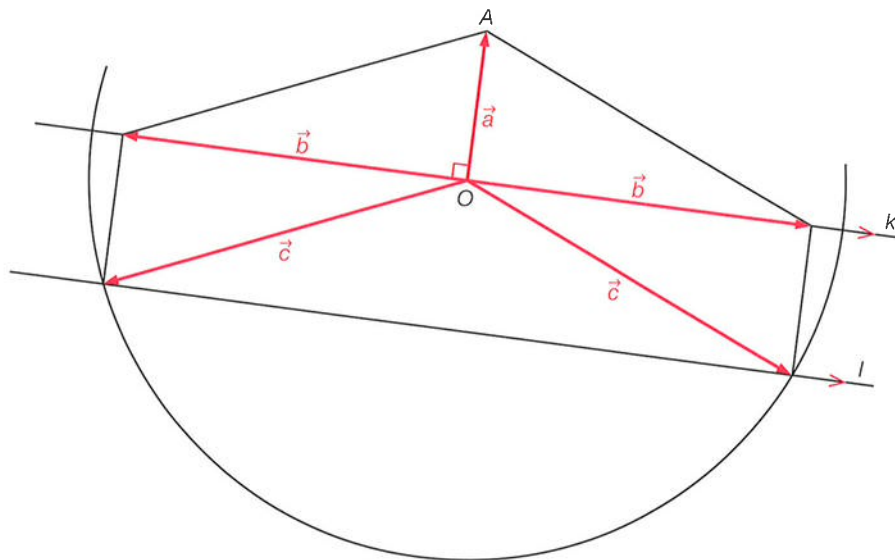
- 2 a** $|\vec{a}| = 8 \sin(17^\circ) \approx 2,34$ en $|\vec{b}| = 8 \cos(17^\circ) \approx 7,65$
- b** $1,5 = |\vec{v}| \cdot \sin(26^\circ)$ geeft $|\vec{v}| = \frac{1,5}{\sin(26^\circ)} \approx 3,42$ en $|\vec{b}| = 3,42 \dots \cos(26^\circ) \approx 3,08$.
- 3 a** $|\vec{a}| = \left| \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right| = \sqrt{4^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17}$
- b** $|\vec{b}| = \left| \begin{pmatrix} \sqrt{3} \\ \sqrt{6} \end{pmatrix} \right| = \sqrt{(\sqrt{3})^2 + (\sqrt{6})^2} = \sqrt{3 + 6} = \sqrt{9} = 3$
- c** $|\vec{c}| = \left| \begin{pmatrix} 0 \\ 5 \end{pmatrix} \right| = \sqrt{0^2 + 5^2} = \sqrt{0 + 25} = \sqrt{25} = 5$
- d** $|\vec{d}| = \left| \begin{pmatrix} -0,3 \\ 0,4 \end{pmatrix} \right| = \sqrt{(-0,3)^2 + 0,4^2} = \sqrt{0,09 + 0,16} = \sqrt{0,25} = 0,5$
- e** $|\vec{e}| = 2 \cdot \left| \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right| = 2 \cdot \sqrt{1^2 + 3^2} = 2 \cdot \sqrt{1 + 9} = 2\sqrt{10}$
- f** $|\vec{f}| = 0,6 \cdot \left| \begin{pmatrix} 6 \\ 8 \end{pmatrix} \right| = 0,6 \cdot \sqrt{6^2 + 8^2} = 0,6 \cdot \sqrt{36 + 64} = 0,6 \cdot \sqrt{100} = 0,6 \cdot 10 = 6$
- 4 a** $\vec{c} = 2\vec{a} + 3\vec{b} = 2 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} + 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \end{pmatrix}$
- b** $\vec{d} = \vec{a} - 2\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 2 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$
- c** $\vec{e} = 4\vec{a} - 3\vec{b} = 4 \cdot \begin{pmatrix} 2 \\ 3 \end{pmatrix} - 3 \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$
- d** $\vec{f} = 2 \cdot (\vec{a} + \vec{b}) = 2 \cdot \left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} \right) = 2 \cdot \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix}$

Bladzijde 57**5 a****b****c**

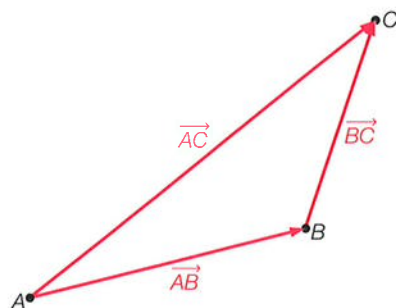
d



- 6 Teken de cirkel met middelpunt O en straal 5.
Teken de lijn l evenwijdig met k , op afstand 2 van k .
De snijpunten van l met de cirkel zijn de eindpunten van de vectoren c .
Met de parallellogrammen krijg je de vectoren b .



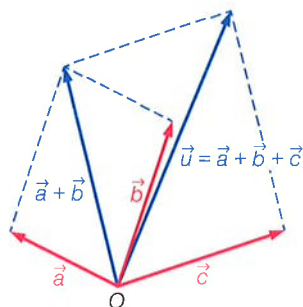
7 a



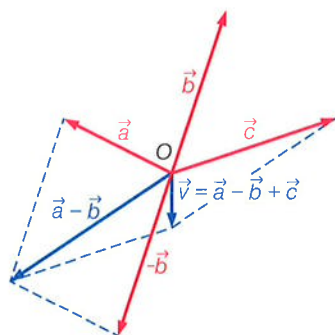
- b $\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$
c $\overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{DF}$
d $\overrightarrow{AK} + \overrightarrow{KL} = \overrightarrow{AL}$
e $\overrightarrow{MN} = \overrightarrow{MP} + \overrightarrow{PN}$

- 8 a** $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$, dus $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{b} - \vec{a}$.
- b** $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \vec{a} + \frac{1}{2}(\vec{b} - \vec{a}) = \vec{a} + \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} = \frac{1}{2}(\vec{a} + \vec{b})$
- c** $\overrightarrow{PQ} = \vec{q} - \vec{p} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$
- $\overrightarrow{QP} = \vec{p} - \vec{q} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$
- $\overrightarrow{OR} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2} \cdot \left(\begin{pmatrix} 7 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 10 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 3\frac{1}{2} \end{pmatrix}$
- $\overrightarrow{QR} = \vec{r} - \vec{q} = \begin{pmatrix} 5 \\ 3\frac{1}{2} \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1\frac{1}{2} \end{pmatrix}$

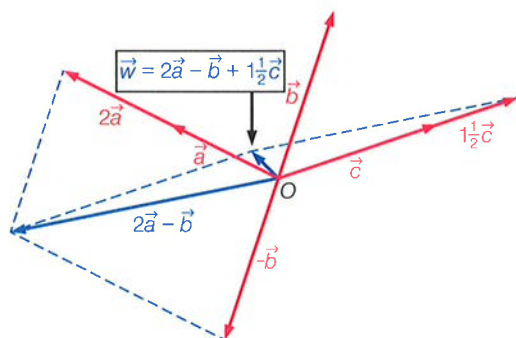
9 a



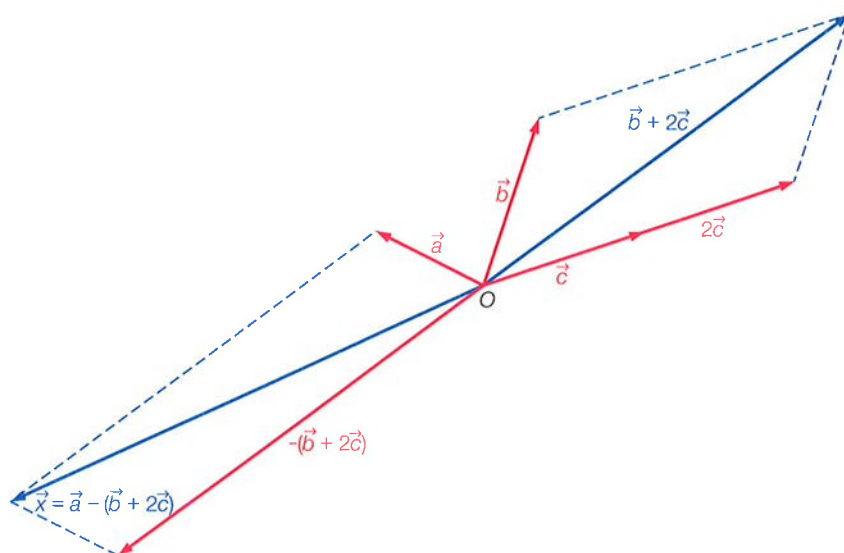
b



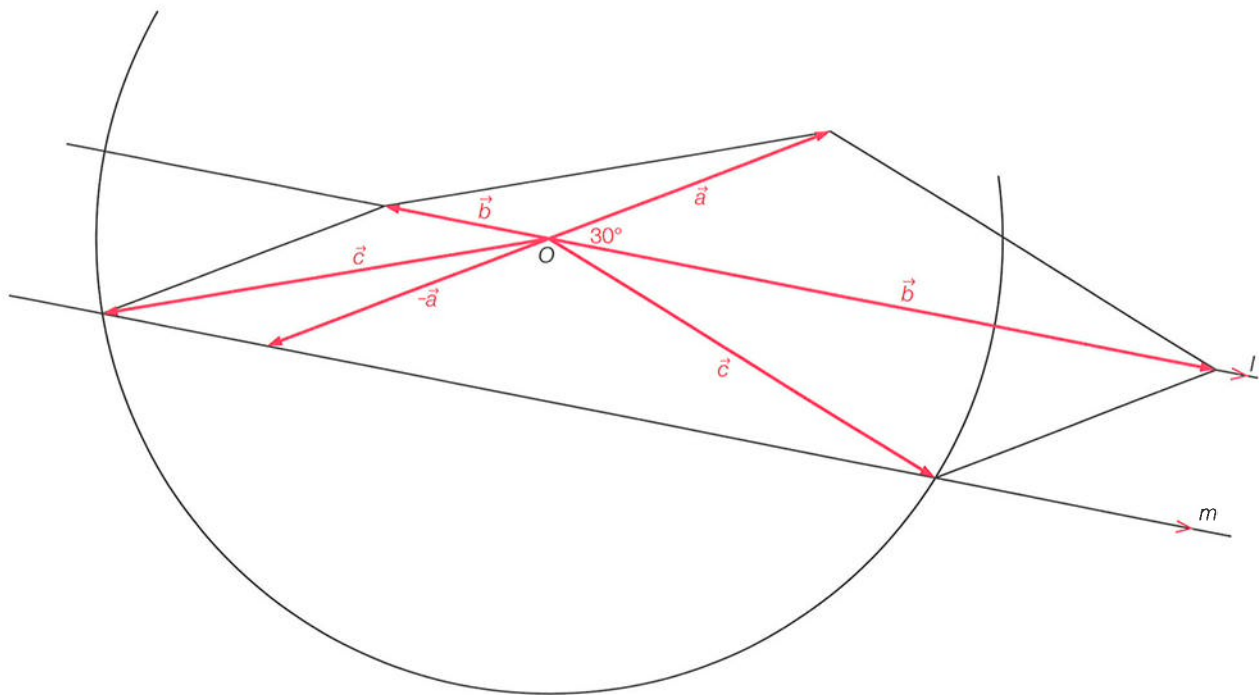
c



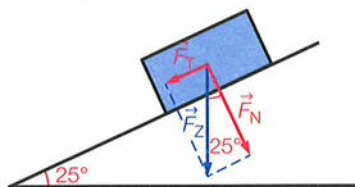
d



10

**Bladzijde 58**

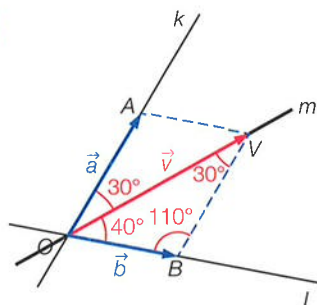
11



De component langs de helling is $F_T = 750 \sin(25^\circ) \approx 317 \text{ N}$.

De component loodrecht op de helling is $F_N = 750 \cos(25^\circ) \approx 680 \text{ N}$.

12



De sinusregel in $\triangle OBV$ geeft $\frac{12}{\sin(110^\circ)} = \frac{|\vec{b}|}{\sin(30^\circ)} = \frac{|\vec{a}|}{\sin(40^\circ)}$.

Dus $|\vec{a}| = \frac{12 \sin(40^\circ)}{\sin(110^\circ)} \approx 8,2$ en $|\vec{b}| = \frac{12 \sin(30^\circ)}{\sin(110^\circ)} \approx 6,4$.

10.2 Vectoren en rotaties**Bladzijde 60**

13

a $\vec{a}_R = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ en $\vec{a}_L = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

b $\vec{b}_R = \begin{pmatrix} q \\ -p \end{pmatrix}$

Bladzijde 61

$$14 \quad \vec{d} = \vec{m} + \overrightarrow{MD} = \vec{m} + \overrightarrow{AM_L} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

Dus $D(-1, 5)$.

Bladzijde 62

$$15 \quad a \quad \vec{b} = \vec{a} + \overrightarrow{AB} = \vec{a} + \overrightarrow{AD_R}$$

$$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \text{ dus } \overrightarrow{AD_R} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}.$$

$$\text{Dit geeft } \vec{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}.$$

Dus $B(9, 5)$.

$$b \quad \vec{n} = \vec{b} + \overrightarrow{BN} = \vec{b} + \overrightarrow{BM_R}$$

$$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BD} = \frac{1}{2}(\vec{d} - \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ 5 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} -7 \\ 1 \end{pmatrix} = \begin{pmatrix} -3\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \text{ dus } \overrightarrow{BM_R} = \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix}.$$

$$\text{Dit geeft } \vec{n} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 3\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 9\frac{1}{2} \\ 8\frac{1}{2} \end{pmatrix}.$$

Dus $N(9\frac{1}{2}, 8\frac{1}{2})$.

$$16 \quad \vec{n} = \frac{1}{2}(\vec{b} + \vec{d}) \text{ met } \vec{d} = \vec{m} + \overrightarrow{MD} = \vec{m} + \overrightarrow{AM_L}$$

$$\vec{m} = \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2}\left(\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\overrightarrow{AM} = \vec{m} - \vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, \text{ dus } \overrightarrow{AM_L} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}.$$

$$\text{Dit geeft } \vec{d} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \text{ en } \vec{n} = \frac{1}{2}\left(\begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix}\right) = \begin{pmatrix} 5\frac{1}{2} \\ 5\frac{1}{2} \end{pmatrix}.$$

Dus $N(5\frac{1}{2}, 5\frac{1}{2})$.

$$17 \quad \vec{q} = \vec{a} + \overrightarrow{AB} + \overrightarrow{BP} + \overrightarrow{PQ} = \vec{a} + \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC} + \frac{1}{3}\overrightarrow{AB} = \vec{a} + 1\frac{1}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{BC}$$

$$\overrightarrow{BC} = \overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \text{ en } \overrightarrow{AB} = \overrightarrow{AD_R} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\vec{q} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + 1\frac{1}{3}\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 + 5\frac{1}{3} - 1 \\ 0 + 4 + 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix}$$

$$\vec{r} = \vec{q} + \frac{1}{3}\overrightarrow{BC} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7\frac{1}{3} \\ 5\frac{1}{3} \end{pmatrix} + \begin{pmatrix} -1 \\ 1\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 6\frac{1}{3} \\ 6\frac{2}{3} \end{pmatrix}$$

Dus $Q(7\frac{1}{3}, 5\frac{1}{3})$ en $R(6\frac{1}{3}, 6\frac{2}{3})$.

$$18 \quad a \quad \vec{p} = \vec{a} + \overrightarrow{AP} = \vec{a} + \overrightarrow{AC_R}$$

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{AD_R}$$

$$\overrightarrow{AD} = \vec{d} - \vec{a} = \begin{pmatrix} 0 \\ d \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ d \end{pmatrix} \text{ en } \overrightarrow{AD_R} = \begin{pmatrix} d \\ a \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} -a \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} -a + d \\ a + d \end{pmatrix} \text{ en } \overrightarrow{AC_R} = \begin{pmatrix} a + d \\ a - d \end{pmatrix}$$

$$\vec{p} = \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} a + d \\ a - d \end{pmatrix} = \begin{pmatrix} 2a + d \\ a - d \end{pmatrix}$$

Dus $P(2a + d, a - d)$.

$$b \quad \vec{q} = \vec{p} + \overrightarrow{PQ} = \vec{p} + \overrightarrow{AC} = \begin{pmatrix} 2a + d \\ a - d \end{pmatrix} + \begin{pmatrix} -a + d \\ a + d \end{pmatrix} = \begin{pmatrix} a + 2d \\ 2a \end{pmatrix}$$

Dus $Q(a + 2d, 2a)$.

Bladzijde 63

19 a $\vec{m} = \frac{1}{2}(\vec{a} + \vec{c})$ met $\vec{c} = \vec{d} + \overrightarrow{DC} = \vec{d} + \overrightarrow{AD}_R$

$$\overrightarrow{AD} = \begin{pmatrix} -a \\ d \end{pmatrix} \text{ en } \overrightarrow{DC} = \overrightarrow{AD}_R = \begin{pmatrix} d \\ a \end{pmatrix}$$

$$\vec{c} = \begin{pmatrix} 0 \\ d \end{pmatrix} + \begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} d \\ a+d \end{pmatrix}$$

$$\vec{m} = \frac{1}{2} \left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} d \\ a+d \end{pmatrix} \right) = \frac{1}{2} \begin{pmatrix} a+d \\ a+d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}d \\ \frac{1}{2}a + \frac{1}{2}d \end{pmatrix}$$

Dus $M(\frac{1}{2}a + \frac{1}{2}d, \frac{1}{2}a + \frac{1}{2}d)$.

Voor alle waarden van a en d is $x_M = y_M$, dus M ligt op de lijn $y = x$.

b $OM = \sqrt{(\frac{1}{2}a + \frac{1}{2}d)^2 + (\frac{1}{2}a + \frac{1}{2}d)^2} = \sqrt{2(\frac{1}{2}a + \frac{1}{2}d)^2} = \sqrt{2(\frac{1}{4}a^2 + \frac{1}{2}ad + \frac{1}{4}d^2)}$
 $= \sqrt{\frac{1}{2}(a^2 + 2ad + d^2)} = \sqrt{\frac{1}{2}(a+d)^2} = \sqrt{\frac{1}{2}} \cdot (a+d) = \frac{1}{2}\sqrt{2} \cdot (a+d)$

20 $\vec{t} = \vec{b} + \overrightarrow{BS} + \overrightarrow{ST} = \vec{b} + \overrightarrow{BA}_L + \overrightarrow{SP}_R$ en $\vec{u} = \vec{t} + \overrightarrow{TU} = \vec{t} + \overrightarrow{SP}$

$$\overrightarrow{BA} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} a-b \\ -c \end{pmatrix}, \text{ dus } \overrightarrow{BA}_L = \begin{pmatrix} c \\ a-b \end{pmatrix}.$$

$$\overrightarrow{SP} = \overrightarrow{BP} - \overrightarrow{BS} = \vec{b}_L - \overrightarrow{BA}_L = \begin{pmatrix} -c \\ b \end{pmatrix} - \begin{pmatrix} c \\ a-b \end{pmatrix} = \begin{pmatrix} -2c \\ -a+2b \end{pmatrix}, \text{ dus } \overrightarrow{SP}_R = \begin{pmatrix} -a+2b \\ 2c \end{pmatrix}.$$

$$\vec{t} = \begin{pmatrix} b \\ c \end{pmatrix} + \begin{pmatrix} c \\ a-b \end{pmatrix} + \begin{pmatrix} -a+2b \\ 2c \end{pmatrix} = \begin{pmatrix} -a+3b+c \\ a-b+3c \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} -a+3b+c \\ a-b+3c \end{pmatrix} + \begin{pmatrix} -2c \\ -a+2b \end{pmatrix} = \begin{pmatrix} -a+3b-c \\ b+3c \end{pmatrix}$$

Dus $T(-a+3b+c, a-b+3c)$ en $U(-a+3b-c, b+3c)$.

21 $\vec{n} = \vec{m} + \overrightarrow{MN} = \vec{m} + \overrightarrow{AB}$

$$\overrightarrow{AD} = \begin{pmatrix} 0 \\ 2a \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ 2a \end{pmatrix} \text{ en } \overrightarrow{AB} = \overrightarrow{AD}_R = \begin{pmatrix} 2a \\ a \end{pmatrix}$$

$$\vec{m} = \frac{1}{2}(\vec{d} + \vec{b}) = \frac{1}{2}(\vec{d} + \vec{a} + \overrightarrow{AB}) = \frac{1}{2} \left(\begin{pmatrix} 0 \\ 2a \end{pmatrix} + \begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} \right) = \frac{1}{2} \begin{pmatrix} 3a \\ 3a \end{pmatrix} = \begin{pmatrix} \frac{3}{2}a \\ \frac{3}{2}a \end{pmatrix}$$

$$\vec{n} = \begin{pmatrix} \frac{3}{2}a \\ \frac{3}{2}a \end{pmatrix} + \begin{pmatrix} 2a \\ a \end{pmatrix} = \begin{pmatrix} \frac{7}{2}a \\ \frac{5}{2}a \end{pmatrix}$$

$$\left. \begin{array}{l} y = mx \\ \text{door } N(3\frac{1}{2}a, 2\frac{1}{2}a) \end{array} \right\} 3\frac{1}{2}am = 2\frac{1}{2}a$$

$$m = \frac{5}{7}$$

Bladzijde 64

22 a $\overrightarrow{AC} = \vec{c} - \vec{a} = \begin{pmatrix} b \\ c \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} b-a \\ c \end{pmatrix}$

$$\vec{d} = \vec{c}_L = \begin{pmatrix} -c \\ b \end{pmatrix} \text{ en } \vec{b} = \vec{a}_L = \begin{pmatrix} 0 \\ a \end{pmatrix} \text{ geeft } \overrightarrow{BD} = \vec{d} - \vec{b} = \begin{pmatrix} -c \\ b \end{pmatrix} - \begin{pmatrix} 0 \\ a \end{pmatrix} = \begin{pmatrix} -c \\ b-a \end{pmatrix}.$$

b Uit vraag a volgt $\overrightarrow{BD} = \overrightarrow{AC}_L$, dus $AC = BD$ en $AC \perp BD$.

23 $M(\frac{1}{2}a, 0)$

$$\vec{p} = \vec{a} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$$

$$\vec{q} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{b}_L = \frac{1}{2}\begin{pmatrix} b \\ c \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -c \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix}$$

$$\overrightarrow{MP} = \vec{p} - \vec{m} = \frac{1}{2}\begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} b+c \\ a-b+c \end{pmatrix}$$

$$\overrightarrow{MQ} = \vec{q} - \vec{m} = \frac{1}{2}\begin{pmatrix} b-c \\ b+c \end{pmatrix} - \begin{pmatrix} \frac{1}{2}a \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -a+b-c \\ b+c \end{pmatrix}$$

$$\overrightarrow{MQ} = \overrightarrow{MP}_L, \text{ dus } MP = MQ \text{ en } MP \perp MQ.$$

Dus $\triangle MPQ$ is een gelijkbenige rechthoekige driehoek.

$$24 \quad \vec{p} = \vec{a} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2} \begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$$

$$\vec{q} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{b}_L = \frac{1}{2} \begin{pmatrix} b \\ c \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -c \\ b \end{pmatrix} = \frac{1}{2} \begin{pmatrix} b-c \\ b+c \end{pmatrix}$$

$$\vec{r} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{a}_R = \frac{1}{2} \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ -a \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix}$$

$$\vec{QR} = \vec{r} - \vec{q} = \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2} \begin{pmatrix} b-c \\ b+c \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a-b+c \\ -a-b-c \end{pmatrix}$$

$$\vec{QR} = \vec{OP}_L, \text{ dus } OP \perp QR.$$

$$\vec{AQ} = \vec{q} - \vec{a} = \frac{1}{2} \begin{pmatrix} b-c \\ b+c \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a+\frac{1}{2}b-\frac{1}{2}c \\ \frac{1}{2}b+\frac{1}{2}c \end{pmatrix}$$

$$\vec{PR} = \vec{r} - \vec{p} = \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}b-\frac{1}{2}c \\ -a+\frac{1}{2}b-\frac{1}{2}c \end{pmatrix}$$

$$\vec{PR} = \vec{AQ}_L, \text{ dus } AQ \perp PR.$$

$$\vec{BR} = \vec{r} - \vec{b} = \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix} - \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a-b \\ -\frac{1}{2}a-c \end{pmatrix}$$

$$\vec{PQ} = \vec{q} - \vec{p} = \frac{1}{2} \begin{pmatrix} b-c \\ b+c \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}a-c \\ -\frac{1}{2}a+b \end{pmatrix}$$

$$\vec{PQ} = \vec{BR}_R, \text{ dus } BR \perp PQ.$$

Bladzijde 65

$$25 \quad \text{a} \quad \vec{p} = \vec{a} + \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AB}_R = \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} b-a \\ c \end{pmatrix} + \frac{1}{2} \begin{pmatrix} c \\ a-b \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix}$$

$$\vec{q} = \vec{c} + \frac{1}{2}\vec{CB} + \frac{1}{2}\vec{CB}_L = \begin{pmatrix} d \\ e \end{pmatrix} + \frac{1}{2} \begin{pmatrix} b-d \\ c-e \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -c+e \\ b-d \end{pmatrix} = \frac{1}{2} \begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix}$$

$$\vec{r} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{c}_L = \frac{1}{2} \begin{pmatrix} d \\ e \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -e \\ d \end{pmatrix} = \frac{1}{2} \begin{pmatrix} d-e \\ d+e \end{pmatrix}$$

$$\vec{s} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{a}_R = \frac{1}{2} \begin{pmatrix} a \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ -a \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix}$$

$$\text{b} \quad \vec{PR} = \vec{r} - \vec{p} = \frac{1}{2} \begin{pmatrix} d-e \\ d+e \end{pmatrix} - \frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -a-b-c+d-e \\ -a+b-c+d+e \end{pmatrix}$$

$$\vec{QS} = \vec{s} - \vec{q} = \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix} - \frac{1}{2} \begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a-b+c-d-e \\ -a-b-c+d+e \end{pmatrix}$$

$$\vec{QS} = \vec{PR}_L, \text{ dus } PR = QS \text{ en } PR \perp QS.$$

$$26 \quad \vec{k} = \frac{1}{2}(\vec{p} + \vec{q}) = \frac{1}{2} \left(\frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} + \frac{1}{2} \begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} \right) = \frac{1}{4} \begin{pmatrix} a+2b+d+e \\ a+2c-d+e \end{pmatrix}$$

$$\vec{l} = \frac{1}{2}(\vec{q} + \vec{r}) = \frac{1}{2} \left(\frac{1}{2} \begin{pmatrix} b-c+d+e \\ b+c-d+e \end{pmatrix} + \frac{1}{2} \begin{pmatrix} d-e \\ d+e \end{pmatrix} \right) = \frac{1}{4} \begin{pmatrix} b-c+2d \\ b+c+2e \end{pmatrix}$$

$$\vec{m} = \frac{1}{2}(\vec{r} + \vec{s}) = \frac{1}{2} \left(\frac{1}{2} \begin{pmatrix} d-e \\ d+e \end{pmatrix} + \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix} \right) = \frac{1}{4} \begin{pmatrix} a+d-e \\ -a+d+e \end{pmatrix}$$

$$\vec{n} = \frac{1}{2}(\vec{p} + \vec{s}) = \frac{1}{2} \left(\frac{1}{2} \begin{pmatrix} a+b+c \\ a-b+c \end{pmatrix} + \frac{1}{2} \begin{pmatrix} a \\ -a \end{pmatrix} \right) = \frac{1}{4} \begin{pmatrix} 2a+b+c \\ -b+c \end{pmatrix}$$

$$\vec{KM} = \vec{m} - \vec{k} = \frac{1}{4} \begin{pmatrix} a+d-e \\ -a+d+e \end{pmatrix} - \frac{1}{4} \begin{pmatrix} a+2b+d+e \\ a+2c-d+e \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -2b-2e \\ -2a-2c+2d \end{pmatrix}$$

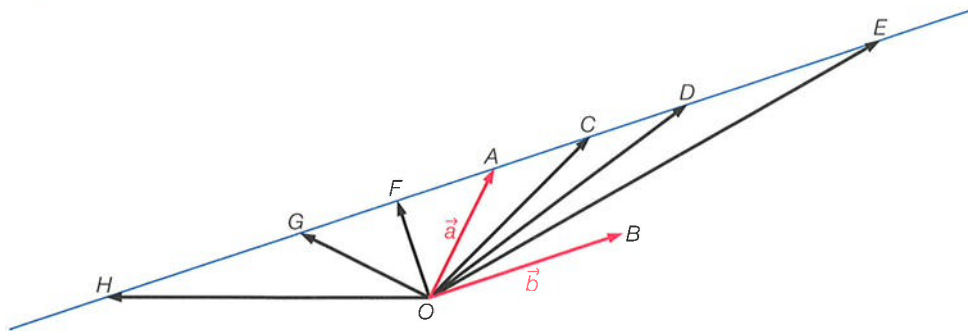
$$\vec{LN} = \vec{n} - \vec{l} = \frac{1}{4} \begin{pmatrix} 2a+b+c \\ -b+c \end{pmatrix} - \frac{1}{4} \begin{pmatrix} b-c+2d \\ b+c+2e \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 2a+2c-2d \\ -2b-2e \end{pmatrix}$$

$$\vec{LN} = \vec{KM}_L, \text{ dus } KM = LN \text{ en } KM \perp LN.$$

10.3 Vectoren en lijnen

Bladzijde 67

27 a



- b De eindpunten van de in a getekende vectoren liggen op de lijn door het eindpunt van de vector $\vec{a}$ en die evenwijdig is met de vector $\vec{b}$.

Bladzijde 69

28 a Je kunt ook $\vec{q}$ als steunvector nemen.

$$\left. \begin{aligned} l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{q} + t \cdot (\vec{q} - \vec{p}) \\ \vec{q} &= \begin{pmatrix} 5 \\ 6 \end{pmatrix} \text{ en } \vec{q} - \vec{p} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \end{aligned} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

- b $\begin{pmatrix} 2 \\ 2 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, dus $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ is ook een richtingsvector van l .

$t = -\frac{1}{2}$ geeft $\begin{pmatrix} 5 \\ 6 \end{pmatrix} + -\frac{1}{2} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, dus $(4, 5)$ is een punt van l en dus is ook $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ een steunvector van l .

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

- c $(3, 4)$ op l .

$x = 3$ geeft $6 + t = 3$, dus $t = -3$.

$t = -3$ en $y = 4$ geeft $p - 3 \cdot 1 = 4$, dus $p = 7$.

29 a $\left. \begin{aligned} k: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{a} + t \cdot (\vec{b} - \vec{a}) \\ \vec{a} &= \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ en } \vec{b} - \vec{a} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix} \triangleq \begin{pmatrix} 6 \\ 1 \end{pmatrix} \end{aligned} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

b $\left. \begin{aligned} l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{c} + t \cdot (\vec{d} - \vec{c}) \\ \vec{c} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ en } \vec{d} - \vec{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \end{aligned} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

c $\left. \begin{aligned} m: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{e} + t \cdot (\vec{f} - \vec{e}) \\ \vec{e} &= \begin{pmatrix} 0 \\ 3 \end{pmatrix} \text{ en } \vec{f} - \vec{e} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \triangleq \begin{pmatrix} 4 \\ 3 \end{pmatrix} \end{aligned} \right\} m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

30 $x = 7$ geeft $-2 + 3t = 7$

$$3t = 9$$

$$t = 3$$

$t = 3$ geeft $y = 5 - 4 \cdot 3 = 5 - 12 = -7$

Dus A ligt op k .

$x = -13$ geeft $-2 + 3t = -13$

$$3t = -11$$

$$t = -3\frac{2}{3}$$

$t = -3\frac{2}{3}$ geeft $y = 5 - 4 \cdot -3\frac{2}{3} = 5 + 14\frac{2}{3} = 19\frac{2}{3}$

Dus B ligt niet op k .

$x = -3\frac{1}{2}$ geeft $-2 + 3t = -3\frac{1}{2}$

$$3t = -1\frac{1}{2}$$

$$t = -\frac{1}{2}$$

$t = -\frac{1}{2}$ geeft $y = 5 - 4 \cdot -\frac{1}{2} = 5 + 2 = 7$

Dus C ligt op k .

31 lijn k

t	0	1
punt	(1, 2)	(4, 3)

lijn l

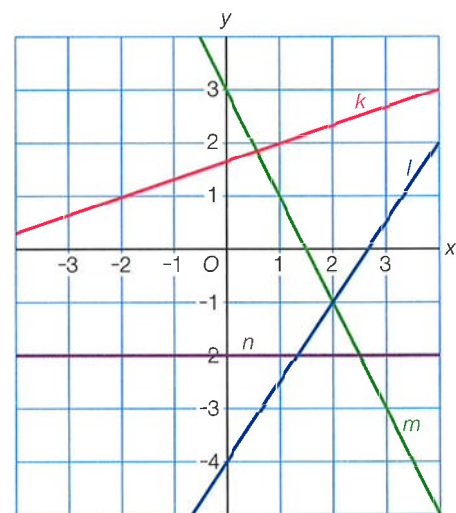
u	0	1
punt	(2, -1)	(4, 2)

lijn m

v	0	1
punt	(0, 3)	(1, 1)

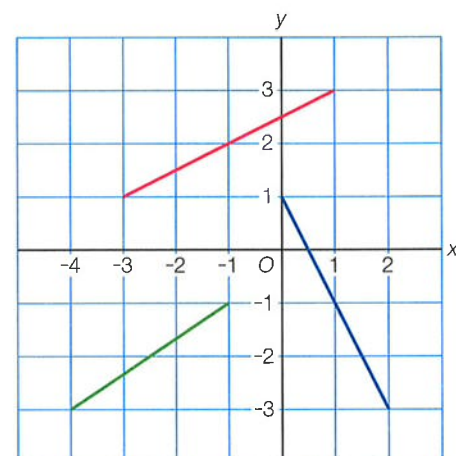
lijn n

w	0	1
punt	(1, -2)	(2, -2)



32

t	-1	1
punt	(-3, 1)	(1, 3)
u	1	3
punt	(0, 1)	(2, -3)
v	-1	0
punt	(-4, -3)	(-1, -1)



33 a $k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{b} - \vec{c})$

$$\left. \begin{array}{l} \vec{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \end{array} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

$$\begin{aligned} \text{b } \vec{m} &= \frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -5 \\ -2 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \\ l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + t \cdot (\vec{b} - \vec{m}) \\ \vec{b} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{m} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 6 \\ 4 \end{pmatrix} \\ \text{c } \vec{n} &= \frac{1}{2}(\vec{a} + \vec{b}) = \frac{1}{2} \cdot \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right) = \frac{1}{2} \cdot \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \\ m: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{n} + t \cdot (\vec{a} - \vec{c}) \\ \vec{n} &= \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + t \cdot \begin{pmatrix} 4 \\ 4 \end{pmatrix} \\ \text{d } &\text{Stel eerst een vectorvoorstelling op van de lijn door } BC. \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + t \cdot (\vec{b} - \vec{c}) \\ \vec{b} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ en } \vec{b} - \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} \left. \vphantom{\begin{pmatrix} x \\ y \end{pmatrix}} \right\} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix} \end{aligned}$$

Dus een vectorvoorstelling van het lijnstuk BC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + t \cdot \begin{pmatrix} 8 \\ 6 \end{pmatrix} \wedge -1 \leq t \leq 0$.

Bladzijde 70

- 34 a** De lijn door de middens van AD en BC is evenwijdig met AB en CD en heeft dus dezelfde richting als AB en CD . Dus een richtingsvector van deze lijn is $\overrightarrow{DC} = \vec{c} - \vec{d}$. De vector vanuit O naar het midden van BC is $\frac{1}{2}(\vec{b} + \vec{c})$. Dus $\frac{1}{2}(\vec{b} + \vec{c})$ is een steunvector van de lijn door de middens van AD en BC . Dus $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}(\vec{b} + \vec{c}) + t \cdot (\vec{c} - \vec{d})$ is een vectorvoorstelling van de lijn door de middens van AD en BC .

$$\begin{aligned} \text{b } \bullet k: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{a} + t \cdot (\vec{b} - \vec{d}) \\ \bullet l: \begin{pmatrix} x \\ y \end{pmatrix} &= \vec{b} + u \cdot \left(\frac{1}{2}(\vec{c} + \vec{d}) - \vec{b} \right) \\ \bullet m: \begin{pmatrix} x \\ y \end{pmatrix} &= \frac{1}{2}(\vec{a} + \vec{c}) + v \cdot (\vec{c} - \vec{b}) \end{aligned}$$

- c** I gaat door B en is evenwijdig met AC .
II gaat door het midden van AB en is evenwijdig met BD .
III gaat door A en het midden van BC .

35 a Substitutie van $x = 1 - 3t$ en $y = 2 + t$ in $y = 4x - 28$ geeft $2 + t = 4(1 - 3t) - 28$
 $2 + t = 4 - 12t - 28$
 $13t = -26$
 $t = -2$

$t = -2$ geeft $x = 1 - 2 \cdot 3 = 7$ en $y = 2 - 2 \cdot 1 = 0$
Dus $S(7, 0)$.

b Substitutie van $x = 7 + 3u$ en $y = -3 + u$ in $2x - 5y = 11$ geeft $2(7 + 3u) - 5(-3 + u) = 11$
 $14 + 6u + 15 - 5u = 11$
 $u = -18$

$u = -18$ geeft $x = 7 - 18 \cdot 3 = -47$ en $y = -3 - 18 \cdot 1 = -21$
Dus $T(-47, -21)$.

36 a Substitutie van $x = 3t$ en $y = 2 + 2t$ in $2x - 5y = 6$ geeft $2 \cdot 3t - 5(2 + 2t) = 6$
 $6t - 10 - 10t = 6$
 $-4t = 16$
 $t = -4$

$t = -4$ geeft $x = -4 \cdot 3 = -12$ en $y = 2 - 4 \cdot 2 = -6$
Dus het snijpunt is $(-12, -6)$.

- b** Substitutie van $x = 2 - u$ en $y = -5 + 3u$ in $3x + 4y = 10$ geeft $3(2 - u) + 4(-5 + 3u) = 10$

$$6 - 3u - 20 + 12u = 10$$

$$9u = 24$$

$$u = 2\frac{2}{3}$$

$$u = 2\frac{2}{3} \text{ geeft } x = 2 + 2\frac{2}{3} \cdot -1 = -\frac{2}{3} \text{ en } y = -5 + 2\frac{2}{3} \cdot 3 = 3$$

Dus het snijpunt is $(-\frac{2}{3}, 3)$.

37 a $\begin{cases} x = 4 + 2t \\ y = 1 - 5t \end{cases} \begin{vmatrix} 5 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 5x = 20 + 10t \\ 2y = 2 - 10t \end{cases} +$

$$\frac{5x + 2y = 22}{5x + 2y = 22} +$$

Dus k : $5x + 2y = 22$.

b $\begin{cases} x = 3 - t \\ y = 2 + 3t \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 3x = 9 - 3t \\ y = 2 + 3t \end{cases} +$

$$\frac{3x + y = 11}{3x + y = 11} +$$

Dus l : $3x + y = 11$.

c $\begin{cases} x = 3t - 1 \\ y = 5t + 1 \end{cases} \begin{vmatrix} 5 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 5x = 15t - 5 \\ 3y = 15t + 3 \end{cases} -$

$$\frac{5x - 3y = -8}{5x - 3y = -8} -$$

Dus m : $5x - 3y = -8$.

Bladzijde 71

- 38 a** Neem je $x = 4t$ in $y = 2x + 3$, dan krijg je $y = 2 \cdot 4t + 3$ oftewel $y = 8t + 3$.

Dus k : $x = 4t \wedge y = 8t + 3$ is een parametervoorstelling van k .

b $x = t + 5$ geeft $y = 2 \cdot (t + 5) + 3$

$$y = 2t + 10 + 3$$

$$y = 2t + 13$$

Dus k : $x = t + 5 \wedge y = 2t + 13$.

c $x = -3t + 2$ geeft $y = 2 \cdot (-3t + 2) + 3$

$$y = -6t + 4 + 3$$

$$y = -6t + 7$$

Dus k : $x = -3t + 2 \wedge y = -6t + 7$.

d $y = 4t$ geeft $4t = 2x + 3$

$$-2x = -4t + 3$$

$$x = 2t - 1\frac{1}{2}$$

Dus k : $x = 2t - 1\frac{1}{2} \wedge y = 4t$.

39 a $\begin{cases} x = 5 + 4t \\ y = 3 - 2t \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix}$ geeft $\begin{cases} x = 5 + 4t \\ 2y = 6 - 4t \end{cases} +$

$$\frac{x + 2y = 11}{x + 2y = 11} +$$

Dus k : $x + 2y = 11$.

- b** Neem $x = 2t$ in $3x - 2y = 6$.

Dit geeft $3 \cdot 2t - 2y = 6$

$$6t - 2y = 6$$

$$-2y = -6t + 6$$

$$y = 3t - 3$$

Dus l : $x = 2t \wedge y = 3t - 3$.

c $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t \cdot \begin{pmatrix} -9 \\ 2 \end{pmatrix}$

d $n: x = 4 + 3t \wedge y = -1 + 2t$

40 a $\begin{cases} x = 5t + p \\ y = 4t + 3 \end{cases} \begin{vmatrix} 4 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 4x = 20t + 4p \\ 5y = 20t + 15 \end{cases} -$

$$\frac{4x - 5y = 4p - 15}{4x - 5y = 4p - 15} -$$

Er moet gelden $4p - 15 = -9$ oftewel $4p = 6$, dus $p = 1\frac{1}{2}$.

$$\text{b } \begin{cases} x = 2t - 1 \\ y = -3t + 5 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 3x = 6t - 3 \\ 2y = -6t + 10 \end{cases} +$$

$$\begin{aligned} 3x + 2y &= 7 \\ 2y &= -3x + 7 \\ y &= -1\frac{1}{2}x + 3\frac{1}{2} \end{aligned}$$

41 a Substitutie van $x = 4 + 2t$ en $y = 5 + 3t$ in $x - 2y = 10$ geeft $4 + 2t - 2(5 + 3t) = 10$

$$\begin{aligned} 4 + 2t - 10 - 6t &= 10 \\ -4t &= 16 \\ t &= -4 \end{aligned}$$

$t = -4$ geeft $x = 4 - 4 \cdot 2 = -4$ en $y = 5 - 4 \cdot 3 = -7$
Dus $S(-4, -7)$.

$$\text{b } \begin{cases} -2 + 3u = -3a + 1 \\ 5 - 4u = 2a \end{cases} \begin{vmatrix} 4 \\ 3 \end{vmatrix} \text{ geeft } \begin{cases} -8 + 12u = -12a + 4 \\ 15 - 12u = 6a \end{cases} +$$

$$\begin{aligned} 7 &= -6a + 4 \\ 6a &= -3 \\ a &= -\frac{1}{2} \end{aligned}$$

Dus $A(-3 \cdot -\frac{1}{2} + 1, 2 \cdot -\frac{1}{2}) = A(2\frac{1}{2}, -1)$.

$$\text{c } \begin{cases} 2v + q = 5 \\ -v - 2q = 2 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 2v + q = 5 \\ -2v - 4q = 4 \end{cases} +$$

$$\begin{aligned} -3q &= 9 \\ q &= -3 \end{aligned}$$

42 a $(3, 4)$ op $l_c: 2x + 5y = c$ geeft $c = 2 \cdot 3 + 5 \cdot 4 = 26$, dus $l: 2x + 5y = 26$.
 $(3, 4)$ op $k_{a,b}: x = at - 3 \wedge y = bt + 1$ geeft $at - 3 = 3$ oftewel $at = 6$ en $bt + 1 = 4$ oftewel $bt = 3$.

Dus $at = 2bt$ oftewel $a = 2b$.

Kies bijvoorbeeld $b = 1$, dan is $a = 2$.

Dit geeft $k: x = 2t - 3 \wedge y = t + 1$.

$$\begin{cases} x = 2t - 3 \\ y = t + 1 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} x = 2t - 3 \\ 2y = 2t + 2 \end{cases} -$$

$$x - 2y = -5$$

Omdat $\frac{2}{1} \neq \frac{5}{-2}$ snijden $k: x - 2y = -5$ en $l: 2x + 5y = 26$ elkaar en vallen ze niet samen.

Dus mogelijke waarden zijn $a = 2$, $b = 1$ en $c = 26$.

$$\text{b } \begin{cases} x = at - 3 \\ y = bt + 1 \end{cases} \begin{vmatrix} b \\ a \end{vmatrix} \text{ geeft } \begin{cases} bx = abt - 3b \\ ay = abt + a \end{cases} -$$

$$bx - ay = -a - 3b$$

$k_{a,b}: bx - ay = -a - 3b$ en $l_c: 2x + 5y = c$ vallen samen als geldt $\frac{b}{2} = \frac{-a}{5} = \frac{-a - 3b}{c}$.

Uit $\frac{b}{2} = \frac{-a}{5}$ volgt $5b = -2a$.

Kies bijvoorbeeld $a = 5$, dan is $b = -2$.

$$\left. \begin{aligned} \frac{-a}{5} &= \frac{-a - 3b}{c} \\ a = 5 \text{ en } b = -2 \end{aligned} \right\} \begin{aligned} \frac{-5}{5} &= \frac{-5 - 3 \cdot -2}{c} \\ \frac{-5}{5} &= \frac{1}{c} \\ c &= -1 \end{aligned}$$

Dus mogelijke waarden zijn $a = 5$, $b = -2$ en $c = -1$.

43 $x = -t - 3 \wedge y = pt - 8$ geeft $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ p \end{pmatrix}$
 $\begin{pmatrix} -1 \\ p \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ geeft $p = -2\frac{1}{2}$
 $(1, q)$ op $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -8 \end{pmatrix} + t \cdot \begin{pmatrix} -1 \\ 2\frac{1}{2} \end{pmatrix}$ geeft $-3 - t = 1$
 $-t = 4$
 $t = -4$
 $t = -4$ geeft $q = -8 - 4 \cdot -2\frac{1}{2} = -8 + 10 = 2$
Dus $p = -2\frac{1}{2}$ en $q = 2$.

10.4 Vectoren en hoeken

Bladzijde 73

44 a $A(a_x, a_y)$, dus $\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$ en $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$.
Kwadrateren geeft $|\vec{a}|^2 = a_x^2 + a_y^2$.
 $B(b_x, b_y)$, dus $\vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix}$ en $|\vec{b}| = \sqrt{b_x^2 + b_y^2}$.
Kwadrateren geeft $|\vec{b}|^2 = b_x^2 + b_y^2$.
b Voor de punten $A(x_A, y_A)$ en $B(x_B, y_B)$ geldt $d(A, B) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$.
Dus voor de punten $A(a_x, a_y)$ en $B(b_x, b_y)$ geldt $d(A, B) = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$.
De lengte van het lijnstuk AB wordt behalve als $d(A, B)$ ook genoteerd als AB , dus
 $AB = \sqrt{(b_x - a_x)^2 + (b_y - a_y)^2}$.
Kwadrateren geeft $AB^2 = (b_x - a_x)^2 + (b_y - a_y)^2$
 $= b_x^2 - 2a_x b_x + a_x^2 + b_y^2 - 2a_y b_y + a_y^2$
 $= a_x^2 + a_y^2 + b_x^2 + b_y^2 - 2a_x b_x - 2a_y b_y$

Bladzijde 75

45 a $\angle(k, l) \approx 180^\circ - 108,4^\circ = 71,6^\circ$
b $\cos(\angle(\vec{r}_m, \vec{r}_n)) = \frac{\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right|} = \frac{2 \cdot -3 + 3 \cdot 1}{\sqrt{13} \cdot \sqrt{10}} = \frac{-3}{\sqrt{130}}$
Dus $\angle(\vec{r}_m, \vec{r}_n) \approx 105,3^\circ$.
46 a $\vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -6 \end{pmatrix} = 2 \cdot -1 + 3 \cdot -6 = -2 - 18 = -20$
b $\vec{c} \cdot \vec{d} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \end{pmatrix} = -1 \cdot 2 + 0 \cdot 4 = -2 + 0 = -2$
c $\vec{e} \cdot \vec{f} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 2 \cdot 3 + -3 \cdot 2 = 6 - 6 = 0$
d $\vec{g} \cdot \vec{h} = \begin{pmatrix} \sqrt{2} \\ \sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 2\sqrt{2} \\ -\sqrt{3} \end{pmatrix} = \sqrt{2} \cdot 2\sqrt{2} + \sqrt{3} \cdot -\sqrt{3} = 4 - 3 = 1$
47 a $\cos(\angle(\vec{a}, \vec{b})) = \frac{\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{3 \cdot 2 + 4 \cdot 1}{\sqrt{25} \cdot \sqrt{5}} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$
Dus $\angle(\vec{a}, \vec{b}) \approx 26,6^\circ$.
b $\cos(\angle(\vec{c}, \vec{d})) = \frac{\begin{pmatrix} 4 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} 4 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right|} = \frac{4 \cdot -1 + -1 \cdot 3}{\sqrt{17} \cdot \sqrt{10}} = \frac{-7}{\sqrt{170}}$
Dus $\angle(\vec{c}, \vec{d}) \approx 125,5^\circ$.

$$\text{c } \cos(\angle(\vec{e}, \vec{f})) = \frac{\begin{pmatrix} 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} 0 \\ 3 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 5 \end{pmatrix} \right|} = \frac{0 \cdot 2 + 3 \cdot 5}{3 \cdot \sqrt{29}} = \frac{15}{3\sqrt{29}} = \frac{5}{\sqrt{29}}$$

$$\text{Dus } \angle(\vec{e}, \vec{f}) \approx 21,8^\circ.$$

$$\text{d } \cos(\angle(\vec{g}, \vec{h})) = \frac{\begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|} = \frac{2 \cdot 5 + (-5) \cdot 2}{\sqrt{29} \cdot \sqrt{29}} = \frac{0}{29} = 0$$

$$\text{Dus } \angle(\vec{g}, \vec{h}) = 90^\circ.$$

$$\text{48 a } \cos(\angle(k, l)) = \frac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{1 \cdot 2 + 2 \cdot 1}{\sqrt{5} \cdot \sqrt{5}} = \frac{4}{5}$$

$$\text{Dus } \angle(k, l) \approx 36,9^\circ.$$

$$\text{b } \cos(\angle(m, n)) = \frac{\begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right|} = \frac{3 \cdot (-1) + 1 \cdot 2}{\sqrt{10} \cdot \sqrt{5}} = \frac{1}{\sqrt{50}}$$

$$\text{Dus } \angle(m, n) \approx 81,9^\circ.$$

$$\text{c } \cos(\angle(p, q)) = \frac{\begin{pmatrix} -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\left| \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|} = \frac{-1 \cdot 2 + 1 \cdot 1}{\sqrt{2} \cdot \sqrt{5}} = \frac{1}{\sqrt{10}}$$

$$\text{Dus } \angle(p, q) \approx 71,6^\circ.$$

Bladzijde 76

$$\text{49 a } \cos(\angle(k, l)) = \frac{\begin{pmatrix} 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right|} = \frac{3 \cdot 3 + (-1) \cdot 5}{\sqrt{10} \cdot \sqrt{34}} = \frac{4}{\sqrt{340}}$$

$$\text{Dus } \angle(k, l) \approx 77,5^\circ.$$

$$\text{b } n: x = 3u - 2 \wedge y = 10u + 3 \text{ oftewel } n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + u \cdot \begin{pmatrix} 3 \\ 10 \end{pmatrix}$$

$$\cos(\angle(m, n)) = \frac{\begin{pmatrix} 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 10 \end{pmatrix}}{\left| \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ 10 \end{pmatrix} \right|} = \frac{2 \cdot 3 + (-1) \cdot 10}{\sqrt{5} \cdot \sqrt{109}} = \frac{4}{\sqrt{545}}$$

$$\text{Dus } \angle(m, n) \approx 80,1^\circ.$$

$$\text{50 a } \overrightarrow{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\cos(\angle(\overrightarrow{BA}, \overrightarrow{BC})) = \frac{\begin{pmatrix} -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right|} = \frac{-2 \cdot (-1) + (-2) \cdot 3}{\sqrt{8} \cdot \sqrt{10}} = \frac{-4}{\sqrt{80}}$$

$$\angle(\overrightarrow{BA}, \overrightarrow{BC}) \approx 116,6^\circ$$

$$\text{Dus } \angle B \approx 116,6^\circ.$$

Alternatieve uitwerking

$$\tan(\alpha) = \text{rc}_{AB} = 1 \text{ geeft } \alpha = 45^\circ$$

$$\tan(\beta) = \text{rc}_{BC} = -3 \text{ geeft } \beta = -71,56\dots^\circ$$

$$\alpha - \beta = 45^\circ - (-71,56\dots^\circ) = 116,56\dots^\circ$$

$$\text{Dus } \angle B \approx 116,6^\circ.$$

$$\begin{aligned} \text{b } \vec{r}_{AD} &= \vec{d} - \vec{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \\ \vec{r}_{BE} &= \vec{e} - \vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix} \\ \cos(\angle(AD, BE)) &= \frac{\left| \begin{pmatrix} -1 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 1 \end{pmatrix} \right|}{\left| \begin{pmatrix} -1 \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -5 \\ 1 \end{pmatrix} \right|} = \frac{|-1 \cdot -5 + 6 \cdot 1|}{\sqrt{37} \cdot \sqrt{26}} = \frac{11}{\sqrt{962}} \\ \angle(AD, BE) &\approx 69,2^\circ \\ \text{Dus de hoek tussen de diagonalen } AD \text{ en } BE &\text{ is } 69,2^\circ. \end{aligned}$$

$$\begin{aligned} \text{51 a } \vec{r}_{AC} &= \vec{c} - \vec{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \\ \vec{r}_{BC} &= \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 4 \end{pmatrix} \\ \cos(\angle(AC, BC)) &= \frac{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 4 \end{pmatrix} \right|} = \frac{|2 \cdot 1 + -5 \cdot 4|}{\sqrt{29} \cdot \sqrt{17}} = \frac{18}{\sqrt{493}} \\ \text{Dus } \angle(AC, BC) &\approx 35,8^\circ. \\ \text{b } \vec{BA} &= \vec{a} - \vec{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \\ \vec{BC} &= \vec{c} - \vec{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \\ \cos(\angle(\vec{BA}, \vec{BC})) &= \frac{\left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|} = \frac{-3 \cdot -1 + 1 \cdot -4}{\sqrt{10} \cdot \sqrt{17}} = \frac{-1}{\sqrt{170}} \\ \angle(\vec{BA}, \vec{BC}) &\approx 94,4^\circ \\ \text{Dus } \angle ABC &\approx 94,4^\circ. \end{aligned}$$

$$\begin{aligned} \text{52 a } S(3, -1) \text{ en } (2, 1) \text{ op } k_{a,b} &\text{ geeft } \vec{r}_{k_{a,b}} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}. \\ \cos(\angle(k, l)) &= \frac{\left| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|} = \frac{|1 \cdot 3 + -2 \cdot -4|}{\sqrt{5} \cdot \sqrt{25}} = \frac{11}{5\sqrt{5}} \\ \text{Dus } \angle(k, l) &\approx 10,3^\circ. \\ \text{b } \cos(\angle(k_{a,1}, l)) &= \frac{\left| \begin{pmatrix} a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} a \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 3 \\ -4 \end{pmatrix} \right|} = \frac{|a \cdot 3 + 1 \cdot -4|}{\sqrt{a^2 + 1} \cdot \sqrt{25}} = \frac{|3a - 4|}{5\sqrt{a^2 + 1}} \\ \text{Voer in } y_1 &= \frac{|3x - 4|}{5\sqrt{x^2 + 1}} \text{ en } y_2 = \cos(80). \\ \text{De optie snijpunt geeft } x &= 0,936... \text{ en } x = 1,973... \\ \text{Dus } a &\approx 0,94 \vee a \approx 1,97. \end{aligned}$$

$$\begin{aligned} \text{53 } \vec{a} \cdot \vec{b} &= \begin{pmatrix} p \\ q \end{pmatrix} \cdot \begin{pmatrix} q \\ -p \end{pmatrix} = pq - pq = 0 \\ \vec{a} \cdot \vec{b} &= 0, \text{ dus } \cos(\angle(\vec{a}, \vec{b})) = 0 \text{ en hieruit volgt } \angle(\vec{a}, \vec{b}) = 90^\circ. \end{aligned}$$

$$\begin{aligned} \text{54 a } \text{Een richtingsvector gaat van het punt } (0, 2) &\text{ naar het punt } (3, 0). \\ \text{Dit is de vector } \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} &= \begin{pmatrix} 3 \\ -2 \end{pmatrix}. \\ \text{Dus } \vec{r}_k &= \begin{pmatrix} 3 \\ -2 \end{pmatrix}. \end{aligned}$$

- b** De vector $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op $\vec{r}_k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$.
 Dus $\vec{n}_k = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ staat loodrecht op de lijn k .
- c** De vector $\vec{n}_l = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$ staat loodrecht op de lijn l .

Bladzijde 78

- 55 a** $\vec{r}_k = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.
 $\left. \begin{array}{l} k: 4x - 3y = c \\ (2, -1) \text{ op } k \end{array} \right\} c = 4 \cdot 2 - 3 \cdot -1 = 11$
 Dus $k: 4x - 3y = 11$.
 $\vec{r}_l = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$.
 $\left. \begin{array}{l} l: 7x + 4y = c \\ (0, 0) \text{ op } l \end{array} \right\} c = 0$
 Dus $l: 7x + 4y = 0$.
- b** $\vec{n}_m = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$, dus $\vec{r}_m = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
 $(-3, 0)$ op m , dus $\vec{s}_m = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$.
 Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
 $\vec{n}_n = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, dus $\vec{r}_n = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.
 $(0, 0)$ op n , dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
 Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = u \cdot \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

Bladzijde 79

- 56 a** $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{n}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
 $\left. \begin{array}{l} l: 5x + 2y = c \\ \text{door } A(4, 1) \end{array} \right\} c = 5 \cdot 4 + 2 \cdot 1 = 22$
 Dus $l: 5x + 2y = 22$.
- b** $n \perp m$, dus $\vec{n}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 $\left. \begin{array}{l} n: 3x + 2y = c \\ \text{door } B(5, -1) \end{array} \right\} c = 3 \cdot 5 + 2 \cdot -1 = 13$
 Dus $n: 3x + 2y = 13$.
- 57 a** $\vec{n}_k = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, dus $\vec{r}_l = \vec{r}_k = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
 $A(5, 2)$ op l , dus $\vec{s}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.
 Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
- b** $n \perp m$, dus $\vec{r}_n = \vec{n}_m = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.
 $B(1, -3)$ op n , dus $\vec{s}_n = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$.
 Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} + u \cdot \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

58 a $\begin{cases} 3+t = -1+3u \\ 3+2t = 5-4u \end{cases}$ oftewel $\begin{cases} t-3u = -4 \\ 2t+4u = 2 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 2t-6u = -8 \\ 2t+4u = 2 \end{cases}$

$$\frac{-10u = -10}{u = 1}$$

$u = 1$ geeft $x = -1 + 3 - 2$ en $y = 5 - 4 - 1$
 Dus $S(2, 1)$.

b m door $A(-3, 5)$ en $B(3, -1)$, dus $\vec{r}_m = \begin{pmatrix} 3-(-3) \\ -1-5 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

m door $A(-3, 5)$, dus $\vec{s}_m = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

n staat loodrecht op $p: -x + 5y = 4$, dus $n: 5x + y = c$.

$n: 5x + y = c$
 door $C(-4, -2)$ $\left. \begin{array}{l} \\ \end{array} \right\} c = 5 \cdot -4 + -2 = -22$

Dus $n: 5x + y = -22$.

Substitutie van $x = -3 + t$ en $y = 5 - t$ in $5x + y = -22$ geeft $5(-3 + t) + 5 - t = -22$

$$\begin{aligned} -15 + 5t + 5 - t &= -22 \\ 4t &= -12 \\ t &= -3 \end{aligned}$$

$t = -3$ geeft $x = -3 + -3 = -6$ en $y = 5 - -3 = 8$

Dus $T(-6, 8)$.

c $\vec{r}_q = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, dus $\vec{n}_q = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

$q: x + 4y = c$
 door $(5, 3)$ $\left. \begin{array}{l} \\ \end{array} \right\} c = 5 + 4 \cdot 3 = 17$

Dus $q: x + 4y = 17$.

Substitutie van $x = 2w - 4$ en $y = w - 6$ in $x + 4y = 17$ geeft $2w - 4 + 4(w - 6) = 17$

$$\begin{aligned} 2w - 4 + 4w - 24 &= 17 \\ 6w &= 45 \\ w &= 7\frac{1}{2} \end{aligned}$$

$w = 7\frac{1}{2}$ geeft $x = 2 \cdot 7\frac{1}{2} - 4 = 11$ en $y = 7\frac{1}{2} - 6 = 1\frac{1}{2}$
 Dus $U(11, 1\frac{1}{2})$.

59 a Snijden van k met de x -as geeft $y = 0$

$$\begin{aligned} 3 + t &= 0 \\ t &= -3 \end{aligned}$$

$t = -3$ geeft $x = -2 + -3 \cdot 5 = -17$, dus $A(-17, 0)$.

$l \perp k$, dus $\vec{n}_l = \vec{r}_k = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$.

$l: 5x + y = c$
 door $A(-17, 0)$ $\left. \begin{array}{l} \\ \end{array} \right\} c = 5 \cdot -17 + 0 = -85$

Dus $l: 5x + y = -85$.

b $\vec{r}_m = \vec{b} - \vec{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$n // m$, dus $\vec{r}_n = \vec{r}_m = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ en $(0, 0)$ op n , dus $\vec{s}_n = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Dus $n: \begin{pmatrix} x \\ y \end{pmatrix} = t \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

c $\vec{r}_q = \vec{e} - \vec{o} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$p \perp q$, dus $\vec{n}_q = \vec{r}_p = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

$q: x - 4y = c$
 door $D(-4, 7)$ $\left. \begin{array}{l} \\ \end{array} \right\} c = -4 - 4 \cdot 7 = -32$

Dus $q: x - 4y = -32$.

60 a $\vec{n}_l = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, dus $\vec{r}_l = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

$$\cos(\angle(k, l)) = \frac{\left| \begin{pmatrix} -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} -4 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 5 \\ 2 \end{pmatrix} \right|} = \frac{|-4 \cdot 5 + 1 \cdot 2|}{\sqrt{17} \cdot \sqrt{29}} = \frac{18}{\sqrt{493}}$$

Dus $\angle(k, l) \approx 35,8^\circ$.

b $\vec{n}_m = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$ en $\vec{n}_n = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

$$\cos(\angle(m, n)) = \frac{\left| \begin{pmatrix} 3 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right|}{\left| \begin{pmatrix} 3 \\ -7 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right|} = \frac{|3 \cdot 1 + -7 \cdot 2|}{\sqrt{58} \cdot \sqrt{5}} = \frac{11}{\sqrt{290}}$$

Dus $\angle(m, n) = \angle(\vec{n}_m, \vec{n}_n) \approx 49,8^\circ$.

c $\tan(\alpha) = \text{rc}_p = 2$ geeft $\alpha = 63,43\dots^\circ$

$\tan(\beta) = \text{rc}_q = -1$ geeft $\beta = -45^\circ$

$\alpha - \beta = 63,43\dots^\circ - -45^\circ = 108,43\dots^\circ$

Dus $\angle(p, q) = 180 - 108,43\dots^\circ \approx 71,6^\circ$.

61 a $k_p \parallel l_p$, dus $\vec{r}_{k_p} = \vec{r}_{l_p} = \begin{pmatrix} 2p \\ p+5 \end{pmatrix}$ en $\vec{n}_{l_p} = \begin{pmatrix} p+5 \\ -2p \end{pmatrix}$.

$l_p: (p+5)x - 2py = c$
door $B(-1, -2)$ $\left. \vphantom{\begin{matrix} l_p: (p+5)x - 2py = c \\ \text{door } B(-1, -2) \end{matrix}} \right\} c = -(p+5) + 4p$

$l_p: (p+5)x - 2py = c$
door $C(p, 4)$ $\left. \vphantom{\begin{matrix} l_p: (p+5)x - 2py = c \\ \text{door } C(p, 4) \end{matrix}} \right\} c = (p+5)p - 8p$

$-(p+5) + 4p = (p+5)p - 8p$

$-p - 5 + 4p = p^2 + 5p - 8p$

$p^2 - 6p + 5 = 0$

$(p-1)(p-5) = 0$

$p = 1 \vee p = 5$

$p = 1$ geeft $l_1: (1+5)x - 2y = -(1+5) + 4$ oftewel $l_1: 6x - 2y = -2$, dus $l_1: 3x - y = -1$.

$p = 5$ geeft $l_5: (5+5)x - 2 \cdot 5y = -(5+5) + 4 \cdot 5$ oftewel $l_5: 10x - 10y = 10$, dus

$l_5: x - y = 1$.

b $D(-2, p-4)$ op $m_p: (p+3)x + (p-1)y = -12$ geeft $(p+3) \cdot -2 + (p-1)(p-4) = -12$

$-2p - 6 + p^2 - 4p - p + 4 = -12$

$p^2 - 7p + 10 = 0$

$(p-2)(p-5) = 0$

$p = 2 \vee p = 5$

$p = 2$ geeft $D(-2, -2)$, $m_2: 5x + y = -12$ en $n_{2,q}: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$.

$m_2 \perp n_{2,q}$, dus $\vec{n}_{m_2} = \vec{r}_{n_{2,q}}$, dus $\begin{pmatrix} 5 \\ 1 \end{pmatrix} = u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$. Dit geeft $5 = qu$ en $1 = u(3-q)$

$1 = 3u - qu$

$1 = 3u - 5$

$3u = 6$

$u = 2$

$5 = qu$ en $u = 2$ geeft $q = 2\frac{1}{2}$.

$p = 5$ geeft $D(-2, 1)$, $m_5: 8x + 4y = -12$ oftewel $m_5: 2x + y = -3$ en $n_{5,q}: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$.

$m_5 \perp n_{5,q}$, dus $\vec{n}_{m_5} = \vec{r}_{n_{5,q}}$, dus $\begin{pmatrix} 2 \\ 1 \end{pmatrix} = u \cdot \begin{pmatrix} q \\ 3-q \end{pmatrix}$. Dit geeft $2 = qu$ en $1 = u(3-q)$

$1 = 3u - qu$

$1 = 3u - 2$

$3u = 3$

$u = 1$

$2 = qu$ en $u = 1$ geeft $q = 2$.

Dus $(p = 2 \wedge q = 2\frac{1}{2}) \vee (p = 5 \wedge q = 2)$.

Bladzijde 80

62 a P op BD , dus $\vec{p} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix}$.

$$\overrightarrow{CP} = \vec{p} - \vec{c} = \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix}$$

b $CP \perp BD$, dus $\overrightarrow{CP} \cdot \vec{r}_{BD} = 0$

$$\begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0$$

$$t-1+9t+12=0$$

$$10t=-11$$

$$t=-1\frac{1}{10}$$

$$t=-1\frac{1}{10} \text{ geeft } P(2-1\frac{1}{10}, 5+3 \cdot -1\frac{1}{10}) = P(\frac{9}{10}, 1\frac{7}{10})$$

c $CP = OP$, dus $|\overrightarrow{CP}| = |\overrightarrow{OP}|$

$$\left| \begin{pmatrix} t-1 \\ 3t+4 \end{pmatrix} \right| = \left| \begin{pmatrix} 2+t \\ 5+3t \end{pmatrix} \right|$$

$$\sqrt{(t-1)^2 + (3t+4)^2} = \sqrt{(2+t)^2 + (5+3t)^2}$$

kwadrateren geeft

$$(t-1)^2 + (3t+4)^2 = (2+t)^2 + (5+3t)^2$$

$$t^2 - 2t + 1 + 9t^2 + 24t + 16 = 4 + 4t + t^2 + 25 + 30t + 9t^2$$

$$-12t = 12$$

$$t = -1$$

$$t = -1 \text{ geeft } P(2 + -1, 5 + 3 \cdot -1) = P(1, 2)$$

63 a $y = 0$ geeft $4x = 12$, dus $x = 3$ en $A(3, 0)$.
 $x = 0$ geeft $3y = 12$, dus $y = 4$ en $B(0, 4)$.

P op l , dus $\vec{p} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix}$.

$$\overrightarrow{AP} = \vec{p} - \vec{a} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix}$$

$$\overrightarrow{BP} = \vec{p} - \vec{b} = \begin{pmatrix} 6+2t \\ -1-3t \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix}$$

$AP \perp BP$, dus $\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$

$$\begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix} \cdot \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix} = 0$$

$$4t^2 + 12t + 6t + 18 + 9t^2 + 15t + 3t + 5 = 0$$

$$13t^2 + 36t + 23 = 0$$

$$D = 36^2 - 4 \cdot 13 \cdot 23 = 100$$

$$t = \frac{-36 \pm 10}{26} = -1 \vee t = \frac{-36 - 10}{26} = -1\frac{10}{13}$$

$$t = -1 \text{ geeft } P(6 + 2 \cdot -1, -1 - 3 \cdot -1) = P(4, 2)$$

$$t = -1\frac{10}{13} \text{ geeft } P(6 + 2 \cdot -1\frac{10}{13}, -1 - 3 \cdot -1\frac{10}{13}) = P(2\frac{6}{13}, 4\frac{4}{13})$$

b $AP = BP$, dus $|\overrightarrow{AP}| = |\overrightarrow{BP}|$

$$\left| \begin{pmatrix} 2t+3 \\ -3t-1 \end{pmatrix} \right| = \left| \begin{pmatrix} 2t+6 \\ -3t-5 \end{pmatrix} \right|$$

$$\sqrt{(2t+3)^2 + (-3t-1)^2} = \sqrt{(2t+6)^2 + (-3t-5)^2}$$

kwadrateren geeft

$$(2t+3)^2 + (-3t-1)^2 = (2t+6)^2 + (-3t-5)^2$$

$$4t^2 + 12t + 9 + 9t^2 + 6t + 1 = 4t^2 + 24t + 36 + 9t^2 + 30t + 25$$

$$-36t = 51$$

$$t = -1\frac{5}{12}$$

$$t = -1\frac{5}{12} \text{ geeft } P(6 + 2 \cdot -1\frac{5}{12}, -1 - 3 \cdot -1\frac{5}{12}) = P(3\frac{1}{6}, 3\frac{1}{4})$$

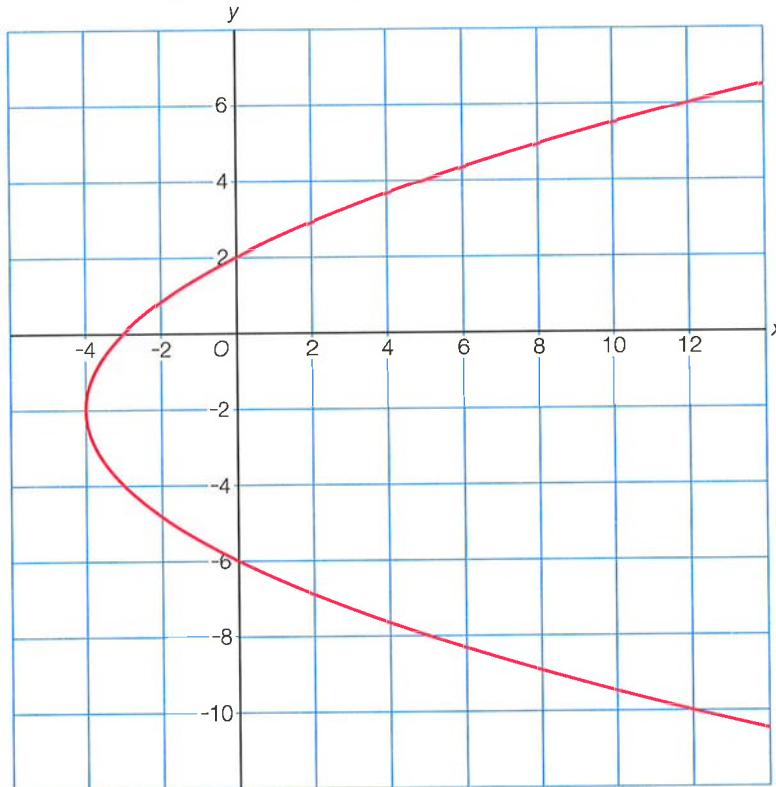
10.5 Vectoren bij snelheid en versnelling

Bladzijde 82

64 a $x(0) = 0 - 0 = 0$ en $y(0) = 0 - 6 = -6$, dus op $t = 0$ is P in het punt $(0, -6)$.

b

t	-2	-1	0	1	2	3	4	5	6
x	12	5	0	-3	-4	-3	0	5	12
y	-10	-8	-6	-4	-2	0	2	4	6



c De verplaatsing van P op $[2, 3]$ is $\begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Bladzijde 85

65 a Evenwijdig met de y -as, dus $x'(t) = 0 \wedge y'(t) \neq 0$
 $t^2 - 4 = 0 \wedge 2t - 2 \neq 0$
 $t^2 = 4 \wedge 2t \neq 2$
 $(t = 2 \vee t = -2) \wedge t \neq 1$
 $t = 2 \vee t = -2$

$$x(2) = \frac{1}{3} \cdot 2^3 - 4 \cdot 2 = -\frac{5}{3} \text{ en } y(2) = 2^2 - 2 \cdot 2 = 0$$

$$x(-2) = \frac{1}{3} \cdot (-2)^3 - 4 \cdot (-2) = \frac{5}{3} \text{ en } y(-2) = (-2)^2 - 2 \cdot (-2) = 8$$

Dus $(-\frac{5}{3}, 0)$ en $(\frac{5}{3}, 8)$.

b Door de oorsprong, dus $x(t) = 0 \wedge y(t) = 0$
 $\frac{1}{3}t^3 - 4t = 0 \wedge t^2 - 2t = 0$
 $\frac{1}{3}t(t^2 - 12) = 0 \wedge t(t - 2) = 0$
 $(t = 0 \vee t^2 = 12) \wedge (t = 0 \vee t = 2)$
 $t = 0$

Stel $l: ax + by = 0$.

$$\vec{r}_l = \begin{pmatrix} x'(0) \\ y'(0) \end{pmatrix} = \begin{pmatrix} 0 - 4 \\ 0 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \triangleq \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \text{ dus } \vec{n}_l = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

Dus $l: x - 2y = 0$.

c Naar links bewegen betekent $x'(t) < 0$ en omhoog bewegen betekent $y'(t) > 0$, dus $x'(t) < 0 \wedge y'(t) > 0$ en dit geeft $t^2 - 4 < 0 \wedge 2t - 2 > 0$.

$$\begin{aligned} \text{d } t^2 - 4 &< 0 \wedge 2t - 2 > 0 \\ t^2 &< 4 \wedge 2t > 2 \\ -2 &< t < 2 \wedge t > 1 \\ 1 &< t < 2 \end{aligned}$$

66 a $x(t) = t^2 - 2$ geeft $x'(t) = 2t$

$$y(t) = t^3 - 4t \text{ geeft } y'(t) = 3t^2 - 4$$

Evenwijdig met de x -as, dus $y'(t) = 0 \wedge x'(t) \neq 0$

$$3t^2 - 4 = 0 \wedge 2t \neq 0$$

$$3t^2 = 4 \wedge t \neq 0$$

$$t^2 = \frac{4}{3} \wedge t \neq 0$$

$$t = \sqrt{\frac{4}{3}} = \frac{2}{3}\sqrt{3} \vee t = -\frac{2}{3}\sqrt{3}$$

$$x(\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3} \text{ en } y(\frac{2}{3}\sqrt{3}) = (\frac{2}{3}\sqrt{3})^3 - 4 \cdot \frac{2}{3}\sqrt{3} = \frac{8}{9}\sqrt{3} - \frac{8}{3}\sqrt{3} = -1\frac{7}{9}\sqrt{3}$$

$$x(-\frac{2}{3}\sqrt{3}) = \frac{4}{3} - 2 = -\frac{2}{3} \text{ en } y(-\frac{2}{3}\sqrt{3}) = (-\frac{2}{3}\sqrt{3})^3 - 4 \cdot (-\frac{2}{3}\sqrt{3}) = -\frac{8}{9}\sqrt{3} + \frac{8}{3}\sqrt{3} = 1\frac{7}{9}\sqrt{3}$$

Dus de punten zijn $(-\frac{2}{3}, -1\frac{7}{9}\sqrt{3})$ en $(-\frac{2}{3}, 1\frac{7}{9}\sqrt{3})$.

b $v(-1) = \sqrt{(x'(-1))^2 + (y'(-1))^2} = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$

c $x'(t) > 0 \wedge y'(t) < 0$ geeft $2t > 0 \wedge 3t^2 - 4 < 0$

$$t > 0 \wedge 3t^2 < 4$$

$$t > 0 \wedge t^2 < \frac{4}{3}$$

$$t > 0 \wedge -\sqrt{\frac{4}{3}} < t < \sqrt{\frac{4}{3}}$$

$$t > 0 \wedge -\frac{2}{3}\sqrt{3} < t < \frac{2}{3}\sqrt{3}$$

$$0 < t < \frac{2}{3}\sqrt{3}$$

d $x(t) = 2$ geeft $t^2 - 2 = 2$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$$y(2) = 2^3 - 4 \cdot 2 = 0 \text{ en } y(-2) = (-2)^3 - 4 \cdot (-2) = 0.$$

$t = 2$ geeft het punt $(2, 0)$ en $t = -2$ geeft het punt $(-2, 0)$, dus de baan snijdt zichzelf in het punt $(2, 0)$.

$$\vec{v}(2) = \begin{pmatrix} x'(2) \\ y'(2) \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\vec{v}(-2) = \begin{pmatrix} x'(-2) \\ y'(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \triangleq \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right|} = \frac{|1 \cdot (-1) + 0 \cdot 0|}{1 \cdot 1} = -1$$

Dus $\varphi \approx 180^\circ$.

67 a $x(t) = 0$ geeft $2t - \frac{1}{6}t^3 = 0$

$$\frac{1}{6}t(12 - t^2) = 0$$

$$t = 0 \vee t^2 = 12$$

$$t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$$

$t = 0$ geeft $(0, 0)$

$$y(2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot 2\sqrt{3} = 3 - 4\sqrt{3}, \text{ dus } B(0, 3 - 4\sqrt{3}) \text{ en}$$

$$y(-2\sqrt{3}) = \frac{1}{4} \cdot 12 - 2 \cdot (-2\sqrt{3}) = 3 + 4\sqrt{3}, \text{ dus } A(0, 3 + 4\sqrt{3}).$$

$$d(A, B) = 3 + 4\sqrt{3} - (3 - 4\sqrt{3}) = 8\sqrt{3}$$

b $x'(t) = 2 - \frac{1}{2}t^2$ en $y'(t) = \frac{1}{2}t - 2$

Evenwijdig met de x -as, dus $y'(t) = 0 \wedge x'(t) \neq 0$

$$\frac{1}{2}t - 2 = 0 \wedge 2 - \frac{1}{2}t^2 \neq 0$$

$$\frac{1}{2}t = 2 \wedge \frac{1}{2}t^2 \neq 2$$

$$t = 4 \wedge t^2 \neq 4$$

$$t = 4 \wedge t \neq 2 \wedge t \neq -2$$

$$t = 4$$

$$x(4) = 2 \cdot 4 - \frac{1}{6} \cdot 4^3 = -\frac{2}{3} \text{ en } y(4) = \frac{1}{4} \cdot 4^2 - 2 \cdot 4 = -4, \text{ dus } (-\frac{2}{3}, -4).$$

Evenwijdig met de y -as, dus $x'(t) = 0 \wedge y'(t) \neq 0$

$$2 - \frac{1}{2}t^2 = 0 \wedge \frac{1}{2}t - 2 \neq 0$$

$$(t = 2 \vee t = -2) \wedge t \neq 4$$

$$t = 2 \vee t = -2$$

$$x(2) = 2 \cdot 2 - \frac{1}{6} \cdot 2^3 = \frac{2}{3} \text{ en } y(2) = \frac{1}{4} \cdot 2^2 - 2 \cdot 2 = -3$$

$$x(-2) = 2 \cdot -2 - \frac{1}{6} \cdot (-2)^3 = -\frac{2}{3} \text{ en } y(-2) = \frac{1}{4} \cdot (-2)^2 - 2 \cdot -2 = 5$$

Dus $(\frac{2}{3}, -3)$ en $(-\frac{2}{3}, 5)$.

c Bij de oorsprong hoort $t = 0$.

$$v(0) = \sqrt{(x'(0))^2 + (y'(0))^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

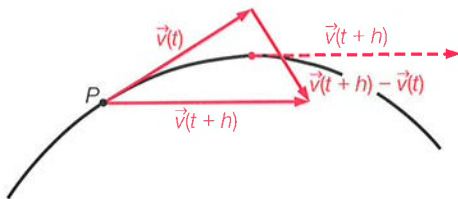
d $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(2 - \frac{1}{2}t^2)^2 + (\frac{1}{2}t - 2)^2}$

$$\text{Voer in } y_1 = \sqrt{(2 - \frac{1}{2}x^2)^2 + (\frac{1}{2}x - 2)^2}.$$

De optie minimum geeft $x = 2,109...$ en $y = 0,971...$

Dus de minimale baansnelheid is ongeveer 0,97 voor $t \approx 2,11$.

68



Bladzijde 86

69

$$\vec{v}(t) = \begin{pmatrix} 2t \\ t^3 - 2 \end{pmatrix} \text{ geeft } \vec{a}(t) = \begin{pmatrix} 2 \\ 3t^2 \end{pmatrix}$$

$$\vec{v}(t) \cdot \vec{a}(t) = \begin{pmatrix} 2t \\ t^3 - 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3t^2 \end{pmatrix} = 4t + 3t^5 - 6t^2 = 3t^5 - 6t^2 + 4t$$

$$|\vec{v}(t)| = \sqrt{t^6 - 4t^3 + 4t^2 + 4}$$

$$\text{Dit geeft } a(t) = \frac{3t^5 - 6t^2 + 4t}{\sqrt{t^6 - 4t^3 + 4t^2 + 4}}.$$

Bladzijde 87

70

a Snijden met de x -as, dus $y(t) = 0$ geeft $t^3 - 3t = 0$

$$t(t^2 - 3) = 0$$

$$t = 0 \vee t^2 = 3$$

$$t = 0 \vee t = \sqrt{3} \vee t = -\sqrt{3}$$

De baan passeert D twee keer, dat is dus op $t = -\sqrt{3}$ en $t = \sqrt{3}$. Dus op $t = 0$ is P in C .

Snijden met de y -as, dus $x(t) = 0$ geeft $t^2 - 4 = 0$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$y(2) = 2^3 - 3 \cdot 2 = 2$, dus de baan passeert A op $t = 2$ en B op $t = -2$.

Dus de punten worden in de volgorde B, D, C, D, A doorlopen.

b $\vec{r}(t) = \begin{pmatrix} t^2 - 4 \\ t^3 - 3t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 2t \\ 3t^2 - 3 \end{pmatrix}$ en

$$v(t) = \sqrt{(2t)^2 + (3t^2 - 3)^2} = \sqrt{4t^2 + 9t^4 - 18t^2 + 9} = \sqrt{9t^4 - 14t^2 + 9}.$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{9t^4 - 14t^2 + 9}} \cdot (36t^3 - 28t) = \frac{18t^3 - 14t}{\sqrt{9t^4 - 14t^2 + 9}}$$

c $v(2) = \sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9} = \sqrt{97}$

$$a(2) = \frac{18 \cdot 2^3 - 14 \cdot 2}{\sqrt{9 \cdot 2^4 - 14 \cdot 2^2 + 9}} = \frac{116}{\sqrt{97}} = 1\frac{19}{97}\sqrt{97}$$

- d** $a(t) = 0$ geeft $18t^3 - 14t = 0$
 $2t(9t^2 - 7) = 0$
 $t = 0 \vee t^2 = \frac{7}{9}$
 $t = 0 \vee t = \sqrt{\frac{7}{9}} \vee t = -\sqrt{\frac{7}{9}}$
 $t = 0 \vee t = \frac{1}{3}\sqrt{7} \vee t = -\frac{1}{3}\sqrt{7}$
 $v(0) = \sqrt{0+9} = 3$
 $v(\frac{1}{3}\sqrt{7}) = \sqrt{9 \cdot (\frac{7}{9})^2 - 14 \cdot \frac{7}{9} + 9} = \sqrt{\frac{32}{9}} = \frac{1}{3}\sqrt{32}$
 $v(-\frac{1}{3}\sqrt{7}) = \sqrt{9 \cdot (\frac{7}{9})^2 - 14 \cdot \frac{7}{9} + 9} = \frac{1}{3}\sqrt{32}$
- e** Voer in $y_1 = \sqrt{9x^4 - 14x^2 + 9}$.
 De optie minimum geeft $y = 1,8856...$ voor $x = -0,8819...$ en $x \approx 0,8819...$
 Dus de minimale baansnelheid is ongeveer 1,886 voor $t \approx -0,882$ en voor $t \approx 0,882$.
- f** $v(t) = 2$ geeft $\sqrt{9t^4 - 14t^2 + 9} = 2$
 kwadrateren geeft
 $9t^4 - 14t^2 + 9 = 4$
 $9t^4 - 14t^2 + 5 = 0$
 Stel $t^2 = u$.
 $9u^2 - 14u + 5 = 0$
 $D = (-14)^2 - 4 \cdot 9 \cdot 5 = 16$
 $u = \frac{14 \pm 4}{18} = 1 \vee \frac{14 - 4}{18} = \frac{5}{9}$
 $t^2 = 1 \vee t^2 = \frac{5}{9}$
 $t = 1 \vee t = -1 \vee t = \frac{\sqrt{5}}{3} \vee t = -\frac{\sqrt{5}}{3}$
 $t = 1 \vee t = -1 \vee t = \frac{1}{3}\sqrt{5} \vee t = -\frac{1}{3}\sqrt{5}$
- g** $t = \sqrt{3}$ geeft $x = (\sqrt{3})^2 - 4 = -1$ en $y = 0$
 $t = -\sqrt{3}$ geeft $x = (-\sqrt{3})^2 - 4 = -1$ en $y = 0$
 Dus de baan snijdt zichzelf in $D(-1, 0)$.
 $\vec{v}(\sqrt{3}) = \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix}$ en $\vec{v}(-\sqrt{3}) = \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix}$.
 $\cos(\varphi) = \frac{\left| \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2\sqrt{3} \\ 6 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -2\sqrt{3} \\ 6 \end{pmatrix} \right|} = \frac{|2\sqrt{3} \cdot -2\sqrt{3} + 6 \cdot 6|}{(\sqrt{12+36})^2} = \frac{24}{48} = \frac{1}{2}$
 Dus $\varphi = 60^\circ$.

Bladzijde 88

- 71 a** Snijden met de x -as, dus $y(t) = 0$ geeft $\frac{1}{2}t^2 - 2t = 0$
 $\frac{1}{2}t(t - 4) = 0$
 $t = 0 \vee t = 4$
 $t = 0$ geeft $(-2, 0)$, dus $t = 0$ hoort bij A en $t = 4$ hoort bij B .
 Dus de punten worden in de volgorde C, A, D, B doorlopen.
- b** $\vec{r}(t) = \begin{pmatrix} \frac{1}{2}t^2 - 2 \\ \frac{1}{2}t^2 - 2t \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} t \\ t - 2 \end{pmatrix}$ en $v(t) = \sqrt{t^2 + (t-2)^2} = \sqrt{t^2 + t^2 - 4t + 4} = \sqrt{2t^2 - 4t + 4}$.
 $a(t) = v'(t) = \frac{1}{2\sqrt{2t^2 - 4t + 4}} \cdot (4t - 4) = \frac{2t - 2}{\sqrt{2t^2 - 4t + 4}}$
- c** $\vec{v}(t) = \begin{pmatrix} t \\ t - 2 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
 $\vec{v}(t) \perp \vec{a}(t)$ geeft $\vec{v}(t) \cdot \vec{a}(t) = 0$
 $\begin{pmatrix} t \\ t - 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$
 $t + t - 2 = 0$
 $2t = 2$
 $t = 1$
 $x(1) = \frac{1}{2} - 2 = -1\frac{1}{2}$ en $y(1) = \frac{1}{2} - 2 = -1\frac{1}{2}$, dus in $(-1\frac{1}{2}, -1\frac{1}{2})$.

d $a(t) = 0$ geeft $2t - 2 = 0$

$$2t = 2$$

$$t = 1$$

$$v(1) = \sqrt{2 - 4 + 4} = \sqrt{2}$$

Dus de minimale baansnelheid is $\sqrt{2}$ voor $t = 1$.

e Bij $t = 4$ hoort B , dus de baansnelheid in B is $v(4) = \sqrt{4^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5}$.

Snijden met de y -as, dus $x(t) = 0$ geeft $\frac{1}{2}t^2 - 2 = 0$

$$\frac{1}{2}t^2 = 2$$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

Bij $t = -2$ hoort dus C en dit geeft $v(-2) = \sqrt{(-2)^2 + (-2 - 2)^2} = \sqrt{20} = 2\sqrt{5}$.

Dus de baansnelheden in B en C zijn gelijk.

f $E(-1\frac{1}{2}, 2\frac{1}{2})$ invullen in $\begin{cases} x(t) = \frac{1}{2}t^2 - 2 \\ y(t) = \frac{1}{2}t^2 - 2t \end{cases}$ geeft $\begin{cases} \frac{1}{2}t^2 - 2 = -1\frac{1}{2} \\ \frac{1}{2}t^2 - 2t = 2\frac{1}{2} \end{cases}$

$$2t - 2 = -4$$

$$2t = -2$$

$$t = -1$$

$$\vec{r}_k = \vec{v}(-1) = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \text{ geeft } \vec{n}_k = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$k: 3x - y = c \quad \left. \begin{array}{l} \text{door } (-1\frac{1}{2}, 2\frac{1}{2}) \end{array} \right\} c = 3 \cdot -1\frac{1}{2} - 2\frac{1}{2} = -7$$

Dus $k: 3x - y = -7$.

72 Q en R op een verticale lijn, dus $x_Q = x_R$.

Dit geeft $\frac{1}{2}t_Q^2 - 2 = \frac{1}{2}t_R^2 - 2$, dus $t_Q = -t_R$.

$$y_R - y_Q = 4 \text{ geeft } \begin{cases} \frac{1}{2}t_R^2 - 2t_R - (\frac{1}{2}t_Q^2 - 2t_Q) = 4 \\ t_Q = -t_R \end{cases} \Rightarrow \begin{cases} \frac{1}{2}t_R^2 - 2t_R - \frac{1}{2}t_R^2 + 2t_R = 4 \\ -4t_R = 4 \end{cases}$$

$$t_R = -1$$

Dus $t_Q = 1$.

$$y(1) = \frac{1}{2} - 2 = -1\frac{1}{2} \text{ en } y(-1) = \frac{1}{2} - 2 = 2\frac{1}{2}.$$

Dus $Q(-1\frac{1}{2}, -1\frac{1}{2})$ en $R(-1\frac{1}{2}, 2\frac{1}{2})$.

73 a $\vec{r}(t) = \begin{pmatrix} 25t \\ -5t^2 + 15t + 3 \end{pmatrix}$ geeft $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$

$$v(0) = \sqrt{25^2 + 15^2} \approx 29,2 \text{ m/s}$$

b Een richtingsvector van een horizontale lijn is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$$\vec{v}(0) = \begin{pmatrix} 25 \\ 15 \end{pmatrix}$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 25 \\ 15 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 25 \\ 15 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{|25 \cdot 1 + 15 \cdot 0|}{\sqrt{850}} = \frac{25}{\sqrt{850}}$$

Dus $\varphi \approx 31^\circ$.

c $y(t) = -5t^2 + 15t + 3$ geeft $y'(t) = -10t + 15$

$$y'(t) = 0 \text{ geeft } -10t + 15 = 0$$

$$-10t = -15$$

$$t = 1,5$$

$$y(1,5) = -5 \cdot 1,5^2 + 15 \cdot 1,5 + 3 = 14,25$$

Dus de maximale hoogte is 14,25 meter.

d $y(t) = 0$ geeft $-5t^2 + 15t + 3 = 0$

$$D = 15^2 - 4 \cdot -5 \cdot 3 = 285$$

$$t = \frac{-15 + \sqrt{285}}{-10} \vee t = \frac{-15 - \sqrt{285}}{-10}$$

$$t = 1,5 - 0,1\sqrt{285} \vee t = 1,5 + 0,1\sqrt{285}$$

vold. niet

$$x(1,5 + 0,1\sqrt{285}) = 25(1,5 + 0,1\sqrt{285}) = 79,70\dots$$

Dus de speer komt 77,7 meter achter de afworplijn neer.

e $\vec{v}(t) = \begin{pmatrix} 25 \\ -10t + 15 \end{pmatrix}$ geeft $\vec{a}(t) = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$

De versnellingsvector is de valversnelling van -10 m/s^2 .

Bladzijde 89

74 a Snijden met de x-as, dus $y(t) = 0$ geeft $t^2 - 4 = 0$

$$t^2 = 4$$

$$t = 2 \vee t = -2$$

$$\vec{r}(t) = \begin{pmatrix} \frac{1}{3}t^3 - 4t \\ t^2 - 4 \end{pmatrix} \text{ geeft } \vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix}$$

$$\vec{v}(2) = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \text{ en } \vec{v}(-2) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}.$$

De snelheidsvectoren in A en B zijn verticaal, dus de raaklijnen aan de baan in A en B zijn verticaal.

b $y(t) = 5$ geeft $t^2 - 4 = 5$

$$t^2 = 9$$

$$t = 3 \vee t = -3$$

$$x(3) = \frac{1}{3} \cdot 3^3 - 4 \cdot 3 = -3, \text{ dus } t = 3 \text{ hoort bij C.}$$

$$\vec{v}(t) = \begin{pmatrix} t^2 - 4 \\ 2t \end{pmatrix} \text{ geeft } v(t) = \sqrt{(t^2 - 4)^2 + (2t)^2} = \sqrt{t^4 - 8t^2 + 16 + 4t^2} = \sqrt{t^4 - 4t^2 + 16}$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{t^4 - 4t^2 + 16}} \cdot (4t^3 - 8t) = \frac{2t^3 - 4t}{\sqrt{t^4 - 4t^2 + 16}}$$

$$\text{De baansnelheid in C is } v(3) = \sqrt{3^4 - 4 \cdot 3^2 + 16} = \sqrt{61}.$$

$$\text{De baanversnelling in C is } a(3) = \frac{2 \cdot 3^3 - 4 \cdot 3}{\sqrt{3^4 - 4 \cdot 3^2 + 16}} = \frac{42}{\sqrt{61}} = \frac{42}{\sqrt{61}} \sqrt{61}.$$

c $v(t) = 6\frac{1}{2}$ geeft $\sqrt{t^4 - 4t^2 + 16} = 6\frac{1}{2}$

kwadrateren geeft

$$t^4 - 4t^2 + 16 = 42\frac{1}{4}$$

$$t^4 - 4t^2 - 26\frac{1}{4} = 0$$

Stel $t^2 = u$.

$$u^2 - 4u - 26\frac{1}{4} = 0$$

$$D = (-4)^2 - 4 \cdot 1 \cdot -26\frac{1}{4} = 121$$

$$u = \frac{4 + 11}{2} = 7\frac{1}{2} \vee u = \frac{4 - 11}{2} = -3\frac{1}{2}$$

$$t^2 = 7\frac{1}{2} \vee t^2 = -3\frac{1}{2}$$

$$t = \sqrt{7\frac{1}{2}} = \frac{1}{2}\sqrt{30} \vee t = -\sqrt{7\frac{1}{2}} = -\frac{1}{2}\sqrt{30}$$

$$x(\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot \frac{1}{2}\sqrt{30} - 4 \cdot \frac{1}{2}\sqrt{30} = \frac{1}{4}\sqrt{30} - 2\sqrt{30} = -\frac{3}{4}\sqrt{30} \text{ en } y(\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}.$$

$$x(-\frac{1}{2}\sqrt{30}) = \frac{1}{3} \cdot 7\frac{1}{2} \cdot -\frac{1}{2}\sqrt{30} - 4 \cdot -\frac{1}{2}\sqrt{30} = -1\frac{1}{4}\sqrt{30} + 2\sqrt{30} = \frac{3}{4}\sqrt{30} \text{ en } y(-\frac{1}{2}\sqrt{30}) = 7\frac{1}{2} - 4 = 3\frac{1}{2}.$$

Dus de punten zijn $(-\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$ en $(\frac{3}{4}\sqrt{30}, 3\frac{1}{2})$.

d Voer in $y_1 = \frac{2x^3 - 4x}{\sqrt{x^4 - 4x^2 + 16}}$.

De optie minimum geeft $x = 0,8510\dots$ en $y = -0,5882\dots$

Dus de baanversnelling heeft voor $t \approx 0,851$ een minimum van ongeveer $-0,588$.

e Snijden met de y -as, dus $x(t) = 0$ geeft $\frac{1}{3}t^3 - 4t = 0$

$$\frac{1}{3}t(t^2 - 12) = 0$$

$$t = 0 \vee t^2 = 12$$

$$t = 0 \vee t = 2\sqrt{3} \vee t = -2\sqrt{3}$$

$$y(2\sqrt{3}) = 12 - 4 = 8 \text{ en } y(-2\sqrt{3}) = 12 - 4 = 8.$$

Dus de baan snijdt zichzelf in $S(0, 8)$.

$$\vec{v}(2\sqrt{3}) = \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \text{ en } \vec{v}(-2\sqrt{3}) = \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix}.$$

$$\cos(\varphi) = \frac{\left| \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix} \right|}{\left| \begin{pmatrix} 8 \\ 4\sqrt{3} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 8 \\ -4\sqrt{3} \end{pmatrix} \right|} = \frac{|8 \cdot 8 + 4\sqrt{3} \cdot -4\sqrt{3}|}{(\sqrt{64 + 48})^2} = \frac{16}{112} = \frac{1}{7}$$

Dus $\varphi \approx 82^\circ$.

Diagnostische toets

Bladzijde 92

1 a $\vec{c} = \vec{a} + 2\vec{b} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$

$$|\vec{c}| = 10$$

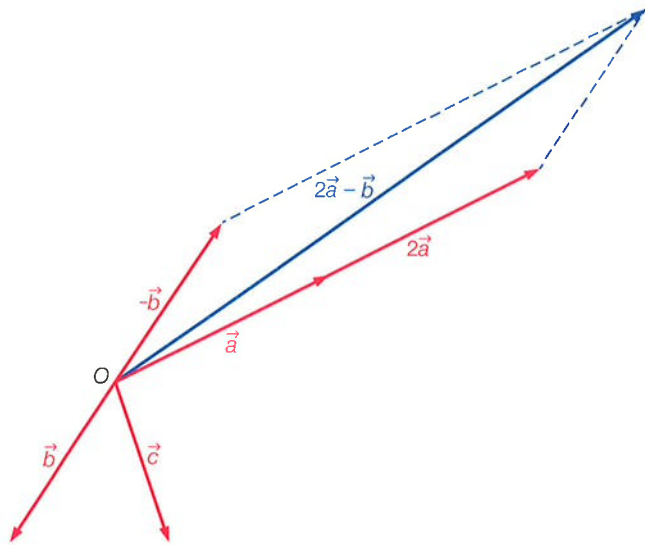
b $\vec{d} = 2\vec{a} - 3\vec{b} = 2 \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} - 3 \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 9 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$

$$|\vec{d}| = \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 5\sqrt{2}$$

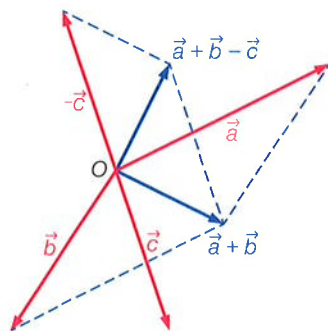
c $\vec{e} = \vec{b} - \frac{1}{2}\vec{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \frac{1}{2} \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$|\vec{e}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

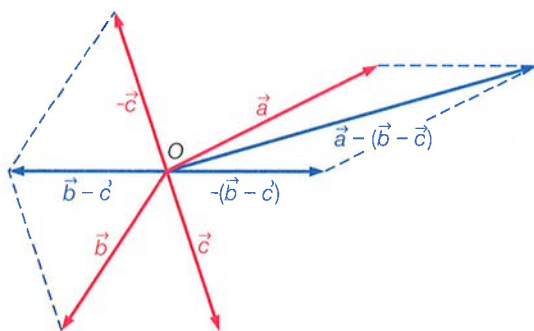
2 a



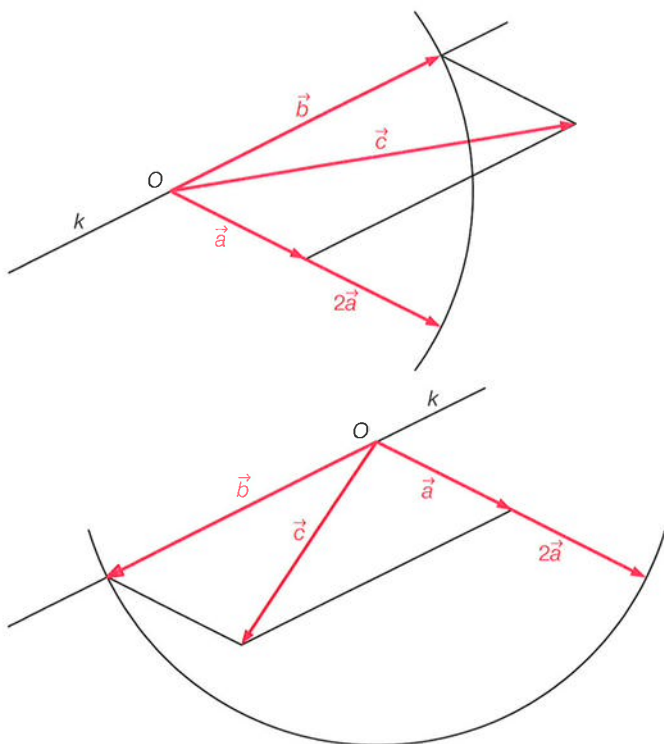
b



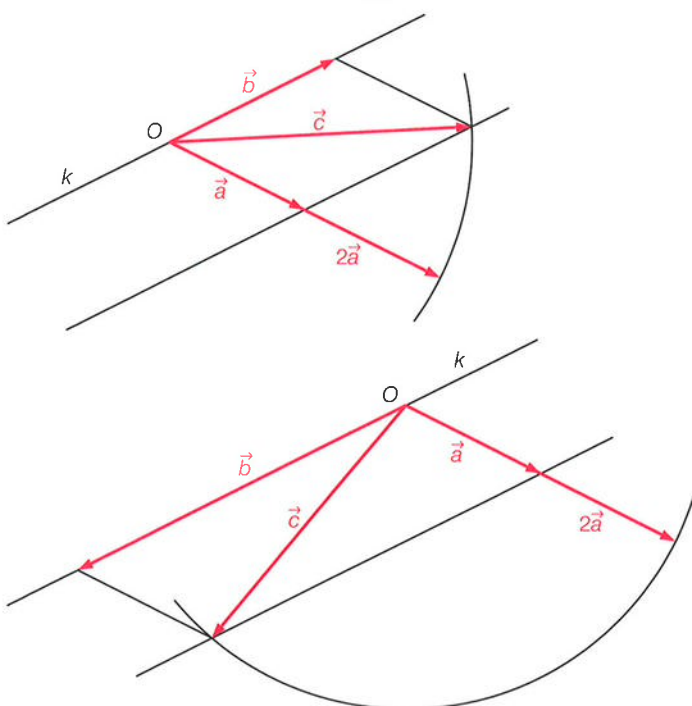
c



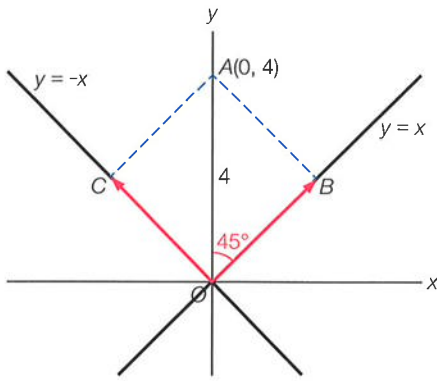
3 a



b



4



In $\triangle OAB$ is $OA = 4$ en $\angle O = 45^\circ$.

$$\text{Dus } OB = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

Ook $OC = 2\sqrt{2}$, dus de lengte van beide componenten is $2\sqrt{2}$.

5

M is het midden van zijde AB .

$$\vec{c} = \vec{b} + \overrightarrow{BC} = \vec{b} + \overrightarrow{MA}_R \text{ en } \vec{d} = \vec{b} + \overrightarrow{AD} = \vec{b} + \overrightarrow{BC}$$

$$M(\frac{1}{2}(4+8), \frac{1}{2}(0+6)) = M(6, 3)$$

$$\overrightarrow{MA} = \vec{a} - \vec{m} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, \text{ dus } \overrightarrow{MA}_R = \begin{pmatrix} -3 \\ 2 \end{pmatrix}.$$

$$\vec{c} = \begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$\vec{d} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Dus $C(5, 8)$ en $D(1, 2)$.

6

$$\overrightarrow{CM} = \vec{m} - \vec{c} = \frac{1}{2}(\vec{a} + \vec{b}) - \vec{c} = \frac{1}{2}\left(\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} b \\ 0 \end{pmatrix}\right) - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b - c \\ -d \end{pmatrix}$$

$$\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{c} + \overrightarrow{CA}_R \text{ en } \vec{h} = \vec{c} + \overrightarrow{CH} = \vec{c} + \overrightarrow{CB}_L$$

$$\overrightarrow{CA} = \vec{a} - \vec{c} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a - c \\ -d \end{pmatrix} \text{ en } \overrightarrow{CB} = \vec{b} - \vec{c} = \begin{pmatrix} b \\ 0 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} b - c \\ -d \end{pmatrix}$$

$$\vec{d} = \vec{c} + \overrightarrow{CD} = \vec{c} + \overrightarrow{CA}_R = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} -d \\ c - a \end{pmatrix} = \begin{pmatrix} c - d \\ -a + c + d \end{pmatrix}$$

$$\vec{h} = \vec{c} + \overrightarrow{CH} = \vec{c} + \overrightarrow{CB}_L = \begin{pmatrix} c \\ d \end{pmatrix} + \begin{pmatrix} d \\ b - c \end{pmatrix} = \begin{pmatrix} c + d \\ b - c + d \end{pmatrix}$$

$$\overrightarrow{DH} = \vec{h} - \vec{d} = \begin{pmatrix} c + d \\ b - c + d \end{pmatrix} - \begin{pmatrix} c - d \\ -a + c + d \end{pmatrix} = \begin{pmatrix} 2d \\ a + b - 2c \end{pmatrix} = 2 \cdot \begin{pmatrix} d \\ \frac{1}{2}a + \frac{1}{2}b - c \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b - c \\ -d \end{pmatrix}_L = \begin{pmatrix} \frac{1}{2}a + \frac{1}{2}b - c \\ d \end{pmatrix}_L, \text{ dus } CM \perp DH \text{ en } DH = 2 \cdot CM.$$

7

$$\text{a } k: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{b} + t \cdot (\vec{a} - \vec{c}) \quad \left. \begin{array}{l} \vec{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \end{array} \right\} k: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

$$\text{b } \vec{m} = \frac{1}{2}(\vec{b} + \vec{c}) = \frac{1}{2}\left(\begin{pmatrix} -4 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -1\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$l: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{m} - \vec{a}) \quad \left. \begin{array}{l} \vec{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ en } \vec{m} - \vec{a} = \begin{pmatrix} -1\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \end{pmatrix} = \begin{pmatrix} -7\frac{1}{2} \\ -\frac{7}{2} \end{pmatrix} \triangleq \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{array} \right\} l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

c Stel eerst een vectorvoorstelling op van de lijn n door A en C .

$$n: \begin{pmatrix} x \\ y \end{pmatrix} = \vec{a} + t \cdot (\vec{a} - \vec{c}) \quad \left. \begin{array}{l} \vec{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \text{ en } \vec{a} - \vec{c} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \end{array} \right\} n: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix}$$

Dus een vectorvoorstelling van het lijnstuk AC is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ 12 \end{pmatrix} \wedge -1 \leq t \leq 0$.

Bladzijde 93

8 Substitutie van $x = -2 + 2t$ en $y = 3 - t$ in $2x - 3y = 22$ geeft $2(-2 + 2t) - 3(3 - t) = 22$
 $-4 + 4t - 9 + 3t = 22$
 $7t = 35$
 $t = 5$

$t = 5$ geeft $x = -2 + 5 \cdot 2 = 8$ en $y = 3 - 5 = -2$, dus $S(8, -2)$.

9 a $k: x = 1 + 3t \wedge y = 2 + 4t$
b $x = t$ geeft $3t - y = 6$ oftewel $y = 3t - 6$.

Dus $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

c $\begin{cases} x = 4 - 2t \\ y = 6 + 3t \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 3x = 12 - 6t \\ 2y = 12 + 6t \end{cases} +$
 $\frac{3x + 2y = 24}{}$

Dus $m: 3x + 2y = 24$.

10 a $2 + t = 2p$ geeft $t = 2p - 2$
 $\begin{cases} 3 - 2t = 3 - p \\ t = 2p - 2 \end{cases} \Rightarrow \begin{cases} 3 - 2(2p - 2) = 3 - p \\ 3 - 4p + 4 = 3 - p \end{cases}$
 $-3p = -4$
 $p = 1\frac{1}{3}$

Dus $A(2 \cdot 1\frac{1}{3}, 3 - 1\frac{1}{3}) = A(2\frac{2}{3}, 1\frac{2}{3})$.

b $3 - 2t = 6$
 $-2t = 3$
 $t = -1\frac{1}{2}$
 $\begin{cases} q + t = 1 \\ t = -1\frac{1}{2} \end{cases} \Rightarrow q = 2\frac{1}{2}$

11 a $\vec{r}_{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$
 $\vec{r}_{BC} = \vec{c} - \vec{b} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -7 \\ -5 \end{pmatrix}$
 $\cos(\angle(AB, BC)) = \frac{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ -5 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -7 \\ -5 \end{pmatrix} \right|} = \frac{|6 \cdot -7 + 1 \cdot -5|}{\sqrt{37} \cdot \sqrt{74}} = \frac{47}{\sqrt{2738}}$

Dus $\angle(AB, BC) \approx 26,1^\circ$.

b $\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ en $\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$
 $\cos(\angle(\vec{AB}, \vec{AC})) = \frac{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|}{\left| \begin{pmatrix} 6 \\ 1 \end{pmatrix} \right| \cdot \left| \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right|} = \frac{-6 - 4}{\sqrt{37} \cdot \sqrt{17}} = \frac{-10}{\sqrt{629}}$

Dus $\angle BAC = \angle(\vec{AB}, \vec{AC}) \approx 113,5^\circ$.

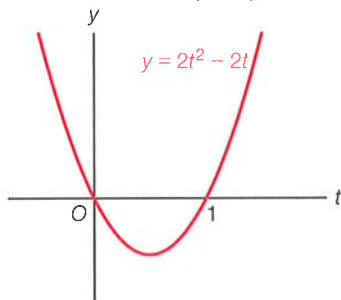
12 a $\vec{r}_k = \vec{r}_l = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$, dus $\vec{n}_k = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.
 $k: 3x + 5y = c$
 door $(3, 4) \Rightarrow c = 3 \cdot 3 + 5 \cdot 4 = 29$
 Dus $k: 3x + 5y = 29$.

b $(1, 1)$ op m , dus $\vec{s}_m = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.
 $m \perp n$, dus $\vec{r}_m = \vec{n}_n = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.
 Dus $m: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

c $p \perp r$, dus $\vec{n}_p = \vec{r}_r = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.
 $p: 4x - y = c$
 door $(-2, 3)$ } $c = 4 \cdot -2 - 3 = -11$
 Dus $p: 4x - y = -11$.

13 Neem $x = 3t$ bij k , dan is $y = \frac{1}{3} \cdot 3t + 3 = t + 3$.
 Dus een parametervoorstelling van k is $k: x = 3t \wedge y = t + 3$.
 $\vec{r}_k = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ en $\vec{r}_l = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ geeft $\vec{r}_k \cdot \vec{r}_l = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} = 3 - 3 = 0$
 Dus $\angle(k, l) = 90^\circ$.

14 a $x(t) = 4t - t^2$ geeft $x'(t) = 4 - 2t$
 $y(t) = \frac{2}{3}t^3 - t^2$ geeft $y'(t) = 2t^2 - 2t$
 $x'(t) > 0$ geeft $4 - 2t > 0$
 $-2t > -4$
 $t < 2$
 $y'(t) > 0$ geeft $2t^2 - 2t > 0$
 $2t(t - 1) > 0$



$2t^2 - 2t > 0$ geeft $t < 0 \vee t > 1$
 $t < 2 \wedge (t < 0 \vee t > 1)$ geeft $t < 0 \vee 1 < t < 2$
 Dus voor $t < 0$ en voor $1 < t < 2$ beweegt P zowel naar rechts als omhoog.
 b $y'(t) = 0$ geeft $t = 0 \vee t = 1$ (zie a) met de bijbehorende punten $(0, 0)$ en $(3, -\frac{1}{3})$.
 $x'(t) = 0$ geeft $t = 2$ (zie a) met het bijbehorende punt $(4, \frac{1}{3})$.
 Dus evenwijdig met de x -as in $(0, 0)$ en $(3, -\frac{1}{3})$ en evenwijdig met de y -as in $(4, \frac{1}{3})$.

c $y(t) = 0$ geeft $\frac{2}{3}t^3 - t^2 = 0$
 $t^2(\frac{2}{3}t - 1) = 0$
 $t = 0 \vee \frac{2}{3}t = 1$
 $t = 0 \vee t = 1\frac{1}{2}$
 vold. niet

$\vec{v}(1\frac{1}{2}) = \begin{pmatrix} x'(1\frac{1}{2}) \\ y'(1\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} 4 - 2 \cdot 1\frac{1}{2} \\ 2 \cdot (1\frac{1}{2})^2 - 2 \cdot 1\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix}$

Bij de x -as hoort richtingsvector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$\cos(\varphi) = \frac{\left| \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 1\frac{1}{2} \end{pmatrix} \right| \cdot \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|} = \frac{1}{\sqrt{3\frac{1}{4}}}$

Dus $\varphi \approx 56,3^\circ$.

d $v(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(4 - 2t)^2 + (2t^2 - 2t)^2}$
 $v(1\frac{1}{2}) = \sqrt{(4 - 3)^2 + (4\frac{1}{2} - 3)^2} = \sqrt{1 + 2\frac{1}{4}} = \sqrt{3\frac{1}{4}} = \frac{1}{2}\sqrt{13}$

e Voer in $y_1 = \sqrt{(4 - 2x)^2 + (2x^2 - 2x)^2}$.

De optie minimum geeft $x = 1,317\dots$ en $y = 1,600\dots$

Dus de minimale baansnelheid van P is ongeveer 1,60 voor $t \approx 1,32$.

$$\mathbf{f} \quad \vec{a}(t) = \begin{pmatrix} x''(t) \\ y''(t) \end{pmatrix} = \begin{pmatrix} -2 \\ 4t-2 \end{pmatrix}$$

$\vec{a}(t)$ is evenwijdig met de x -as geeft $y''(t) = 0$

$$4t - 2 = 0$$

$$4t = 2$$

$$t = \frac{1}{2}$$

$$x\left(\frac{1}{2}\right) = 4 \cdot \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 1\frac{3}{4} \text{ en } y\left(\frac{1}{2}\right) = \frac{2}{3} \cdot \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 = -\frac{1}{6}$$

Dus in het punt $(1\frac{3}{4}, -\frac{1}{6})$.

$$\mathbf{g} \quad v(t) = \sqrt{(4-2t)^2 + (2t^2-2t)^2} = \sqrt{16-16t+4t^2+4t^4-8t^3+4t^2} = \sqrt{4t^4-8t^3+8t^2-16t+16}$$

$$a(t) = v'(t) = \frac{1}{2\sqrt{4t^4-8t^3+8t^2-16t+16}} \cdot (16t^3-24t^2+16t-16) = \frac{8t^3-12t^2+8t-8}{\sqrt{4t^4-8t^3+8t^2-16t+16}}$$

$$a(3) = \frac{216-108+24-8}{\sqrt{324-216+72-48+16}} = \frac{124}{\sqrt{148}} = \frac{124}{2\sqrt{37}} = 1\frac{25}{37}\sqrt{37}$$